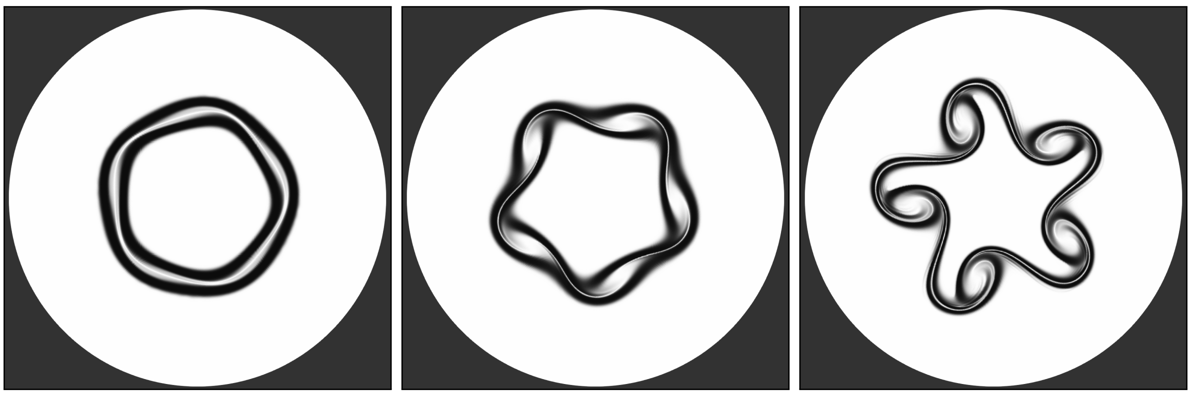
*Texas Tech Department of Mathematics & Statistics*

# A new graduate-level course this Spring 2024: Numerical Analysis of Partial Differential Equations, Part I MATH 5344

Instructor: Ignacio Tomas, igtomas@ttu.edu
https://www.math.ttu.edu/~igtomas/



**Cold-plasma Diocotron instability.** This sequence of snapshots was extracted from the instructor's research. It illustrates the solution of the Euler-Poisson model with non-zero magnetic fields. It shows a rotational instability known as Diocotron instability. The initial data for this problem is $\theta$-periodic, it describes a rotating electron-gas in a state of force balance: pressure, centrifugal and electrostatic forces push the electron gas radially outwards, while the magnetic force pushes inwards. Such equilibrium profile is not stable under perturbation and develops the pattern shown in this sequence.

## 1. Motivations and objectives

For many PDE models of Applied Mechanics (i.e. fluid and solid mechanics) there are numerical methods that do a decent job. However, once we step into the world of coupled problems, nonlinear problems and, broadly speaking, computational physics, the situation is drastically different. It is very easy to find large families of PDEs for which either "nothing works, or everything works, but nothing works particularly well"[1]. The goal of the course is to develop basic mathematical understanding required to develop an analyze numerical methods for PDEs. In general terms: if a method works, we should be able to explain why, and if it doesn't we should be able to explain it as well. This will be a course sequence (Part I and II) that should lay out the fundamentals of Numerical Analysis of Partial Differential equations. This is intended to be basic preparatory training for research.

## 2. Audience

This course welcomes the presence of Math and non-Math majors. Engineering, Physics and Chemistry majors are encouraged to take this class. The mathematical level of the class will be adjusted accordingly depending on the pool of students and their background. In particular, the class should be of direct interest to students whose main goal is to develop a PhD dissertation on Numerical Analysis. More generally, the class should be of interest to any graduate student whose PhD work includes numerical solution of PDE models that cannot be solved using commercial packages.

## 3. Expected Background

Some knowledge of Real Analysis, Functional Analysis and PDEs, is very much welcome. However, we will carry out a light review of most pre-requisite background.

[1]The quote is attributed to Philip Roe.

## 4. Contents and Bibliography

Part I will cover mostly elliptic and stationary hyperbolic problems during Spring 2024. Part II will make an emphasis on time-dependent parabolic and hyperbolic problems during Fall 2024. This course does NOT intend to be a replacement of a graduate course sequence on PDEs. However, we will cover the rudiments of classical/strong solutions as well as modern PDE theory (e.g. weak solutions in the context of Sobolev spaces) at the level of the book of L. Evans. This will help level the class and accommodate all backgrounds.

Part I we will cover the following:

- *Rudiments of Functional Analysis:* Hilbert and Banach spaces, Multi-d integration by parts, Weakly differentiable functions, Sobolev Spaces, Lipschitz and Holder spaces, Sobolev embeddings and strong-compactness, Poincare's and trace inequalities.
- *Well-posedness of linear problems in Banach spaces:* Banach-Nečas-Babuška theorem, Coercivity and Lax-Milgram theorem, Cea's and related quasi-optimality lemmas, Abstract saddle-point problems.
- *Interpolation:* Polynomial interpolation, Simplicial and tensor product finite elements, Inverse inequalities, Bramble-Hilbert lemma, Interpolation operators.
- *Galerkin approximation:* Elliptic PDEs, Conforming approximation, Nitsche-like boundary conditions, Interior penalty approximation, M-matrices and max/min principles.
- *Mixed problems:* Mixed Laplacian, Stokes, Eddy-Currents, Stable-pairs, Fortin's lemma, Linear algebra of mixed problems.
- *Non-galerkin methods:* Finite difference methods, Finite volume methods, Poisson problem in 1d, L-infinity estimates (barrier functions, maximum and mimimum principles).
- *First-order stationary PDEs:* Galerkin Least-Squares, Discontiuous Galerkin.

The lectures will be prepared using selected chapters/material from:

- J-L. Guermond and A. Ern, Theory and Practice of Finite Elements, 2004.
- A. Ern and D. DiPietro, Mathematical Aspects of Discontinuous Galerkin Methods, 2011.
- A. Salgado and S. Wise, Classical Numerical Analysis: A Comprehensive Course, 2023
- S. Larrson and V. Thomee, Partial Differential Equations with Numerical Methods, 2005
- E. Godlewski and P-A. Raviart, Numerical Approximation of Hyperbolic Systems of Conservation Laws
- Lawrence C. Evans, Partial Differential Equations, 2022.

## 5. Evaluation

The course will have a total of 6-9 homework assignments. The homework has to be turned in by its deadline, which will be about two weeks after its assignment. The homework will be composed of 80% theory and 20% coding using Matlab or Python. Alternatively, depending on the pool of students and their interests, the instructor might dedicate a few classes to teaching a much more powerful framework provided by the library `deal.ii`. Final examination is oral: it will consist of a review, extension and/or modification of the problems considered in the homework and their discussion on the blackboard.

## 6. Registration

The course will be officially offered as "MATH 5344, Topics in Numerical Analysis I". The corresponding continuation will be called "MATH 5345, Topics in Numerical Analysis II".

## 7. Questions

Feel free to contact the instructor at igtomas@ttu.edu, or visiting his office MATH221.