Supplemental Separable Equations

1. $y' = \dfrac{1-x^2}{y^2} \quad \Rightarrow \quad y^2 dy = (1-x^2)dx \quad \Rightarrow \quad y = (3x - x^3 + k)^{1/3}$

2. $y' = 3yx^2 \quad \Rightarrow \quad \dfrac{dy}{y} = 2x^2 dx \quad \Rightarrow \quad y = ke^{x^3}$

3. $xy' = \dfrac{1-y^2}{2y} \quad \Rightarrow \quad \dfrac{2y}{y^2-1} = -\dfrac{dx}{x} \quad \Rightarrow \quad y = \pm(1 + kx^{-1})^{1/2}$

4. $y' = \dfrac{\cos^2(y)x}{1+x^2} \quad \Rightarrow \quad \sec^2(y)dy = \dfrac{xdx}{1+x^2} \quad \Rightarrow \quad y = \tan^{-1}(\ln(1+x^2)^{1/2} + k)$

5. $y' = 4x^3(1-y), \;\; y(0) = 3 \quad \Rightarrow \quad \dfrac{dy}{1-y} = 4x^3 dx \quad \Rightarrow \;\; y = ke^{x^4} + 1, \;\; y = 2e^{x^4} + 1$

6. $y' = 2\sqrt{y+1}\cos(x), \;\; y(\pi) = 0 \;\Rightarrow 1/2(y+1)^{-1/2}dy = \cos(x)dx \quad \Rightarrow$
   $(y+1)^{1/2} = \sin(x) + C \quad \Rightarrow \quad y = (\sin(x) + k)^2 - 1, \;\; y = \sin^2(x) + 2\sin(x)$

7. $y' = \dfrac{3x^2+4x+2}{2y+1}, \quad y(0) = -1 \quad \Rightarrow \quad (2y+1)dy = (3x^2+4x+2)dx \quad \Rightarrow$
   $y^2 + y = x^3 + 2x^2 + 2x + C, \;\; y^2 + y = x^3 + 2x^2 + 2x$

8. $y' = 2x\sin^2(y), \quad y(0) = \pi/4 \quad \Rightarrow \quad \csc^2(y)dy = 2xdx \quad \Rightarrow \quad -\cot(y) =$
   $x^2 + C, \;\; y = \cot^{-1}(1-x^2)$

9. $\sqrt{1-y^2}dx = \sqrt{1-x^2}dy, \quad y(0) = \sqrt{3}2 \quad \Rightarrow \quad \dfrac{dy}{\sqrt{1-y^2}} = \dfrac{dx}{\sqrt{1-x^2}} \quad \Rightarrow$
   $\sin^{-1}(y) = \sin^{-1}(x) + C \;\Rightarrow\; y = \sin(\sin^{-1}(x) + C \;\Rightarrow\; y = \sin(\sin^{-1}(x) + \pi/3)$

10. $\dfrac{dx}{dt} = 4(x^2+1), \;\; x(\pi/4) = 1 \quad \Rightarrow \quad \dfrac{dx}{(x^2+1)} = 4dt \quad \Rightarrow \quad \tan^{-1}(x) = 4t + C$
    $\Rightarrow \quad x = \tan(4t + C) \quad \Rightarrow \quad x = \tan(4t + \pi/4)$

11. $x^2y' = y - xy, \;\; y(-1) = -1 \;\Rightarrow\; \dfrac{dy}{y} = \dfrac{1-x}{x^2} \;\Rightarrow\; \ln(|y|) = (-x - \ln(|x|)) + C$
    $\Rightarrow \quad -y = e^{-x-\ln(|x|)+C} = \dfrac{ke^{x^{-1}}}{-x} \quad \Rightarrow \quad y = \dfrac{e^{x^{-1}}}{ex}$