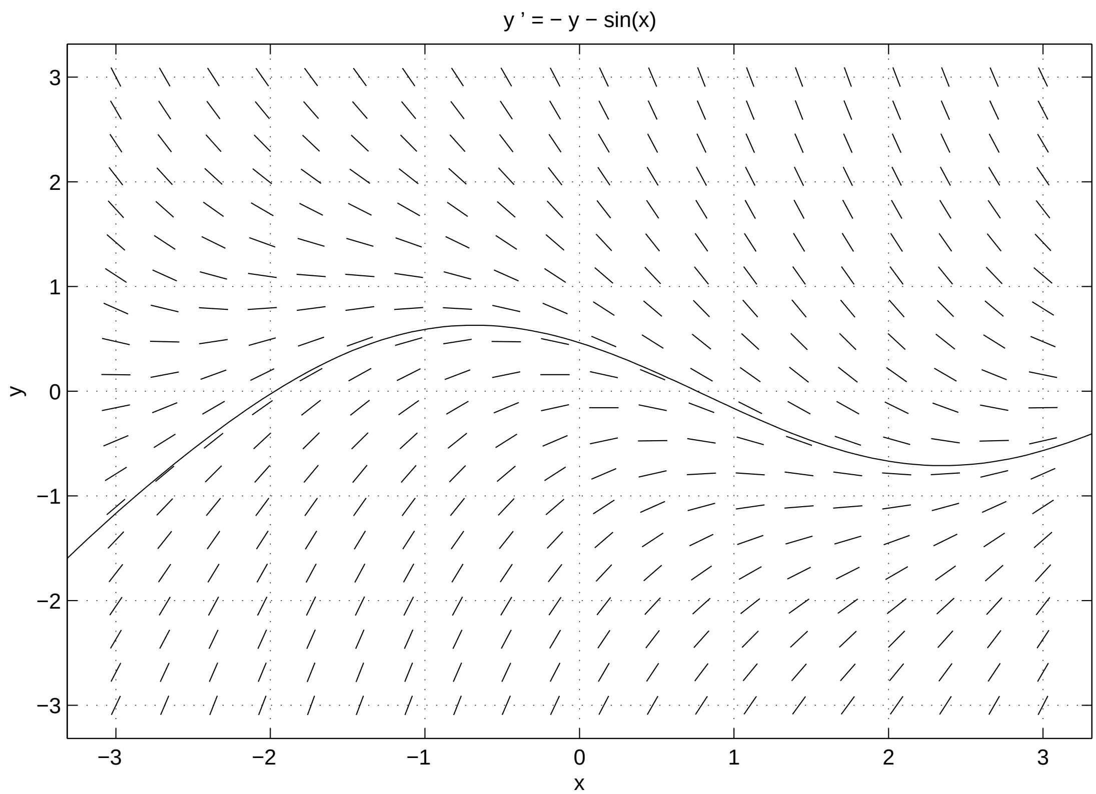
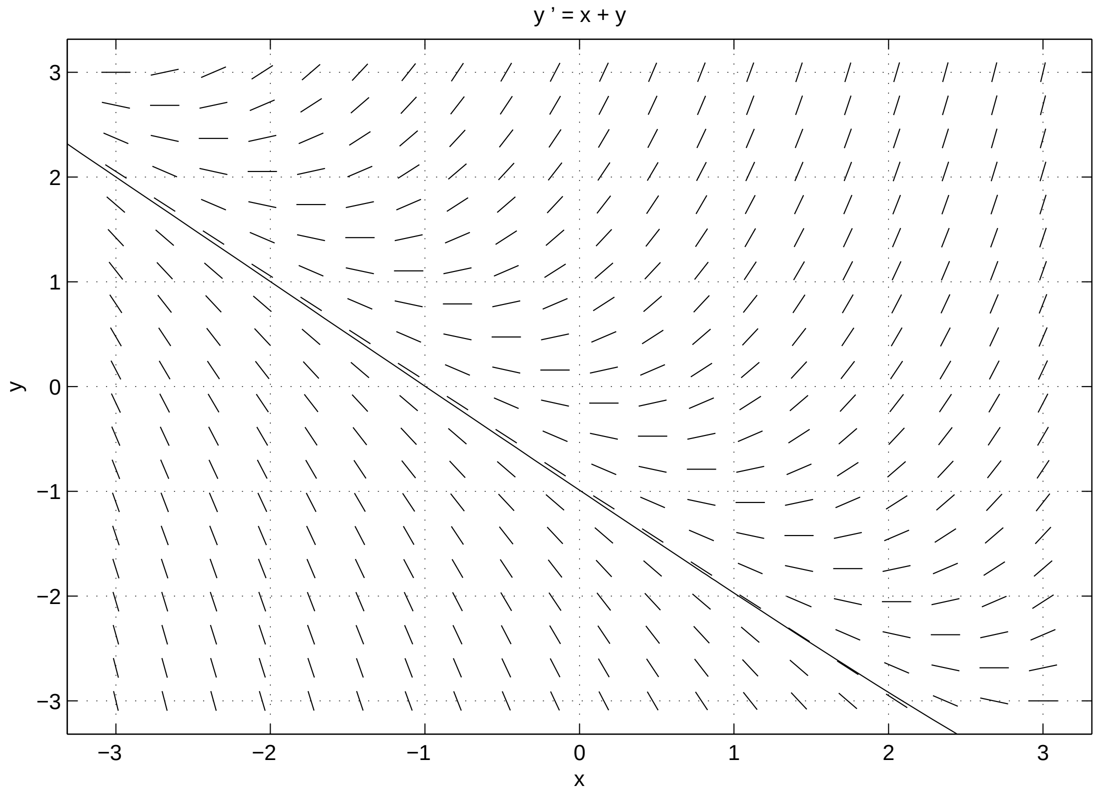
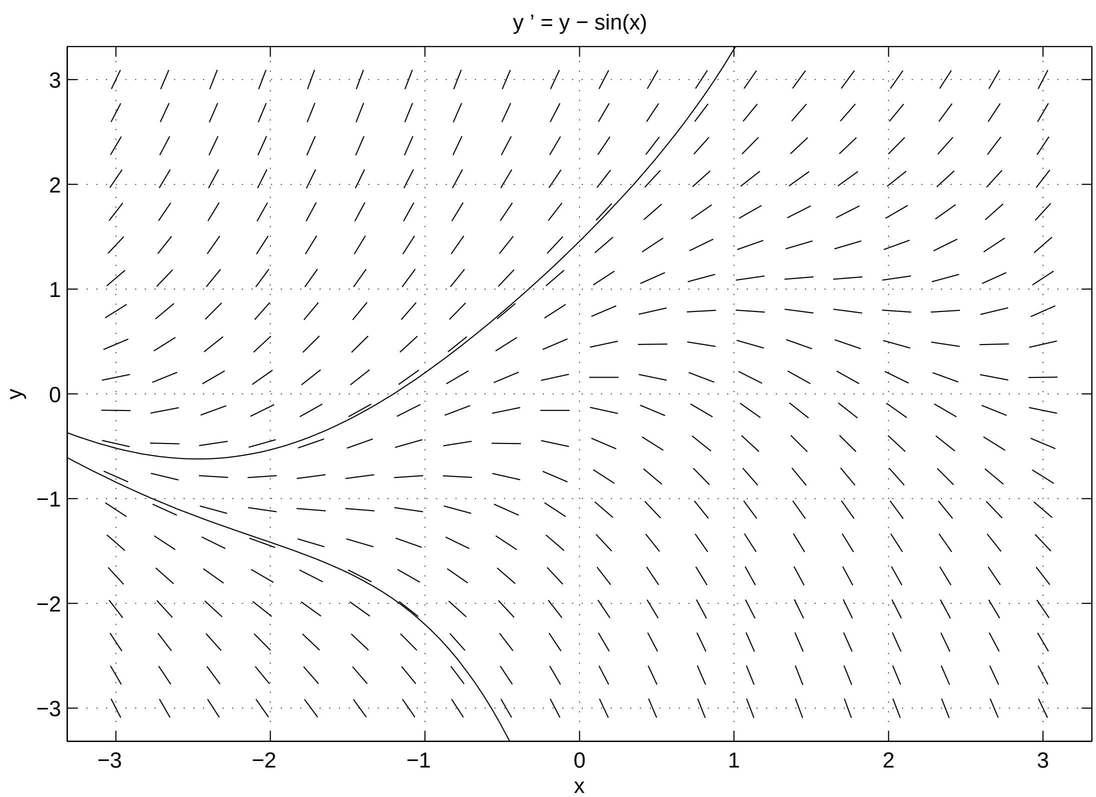
Direction Fields

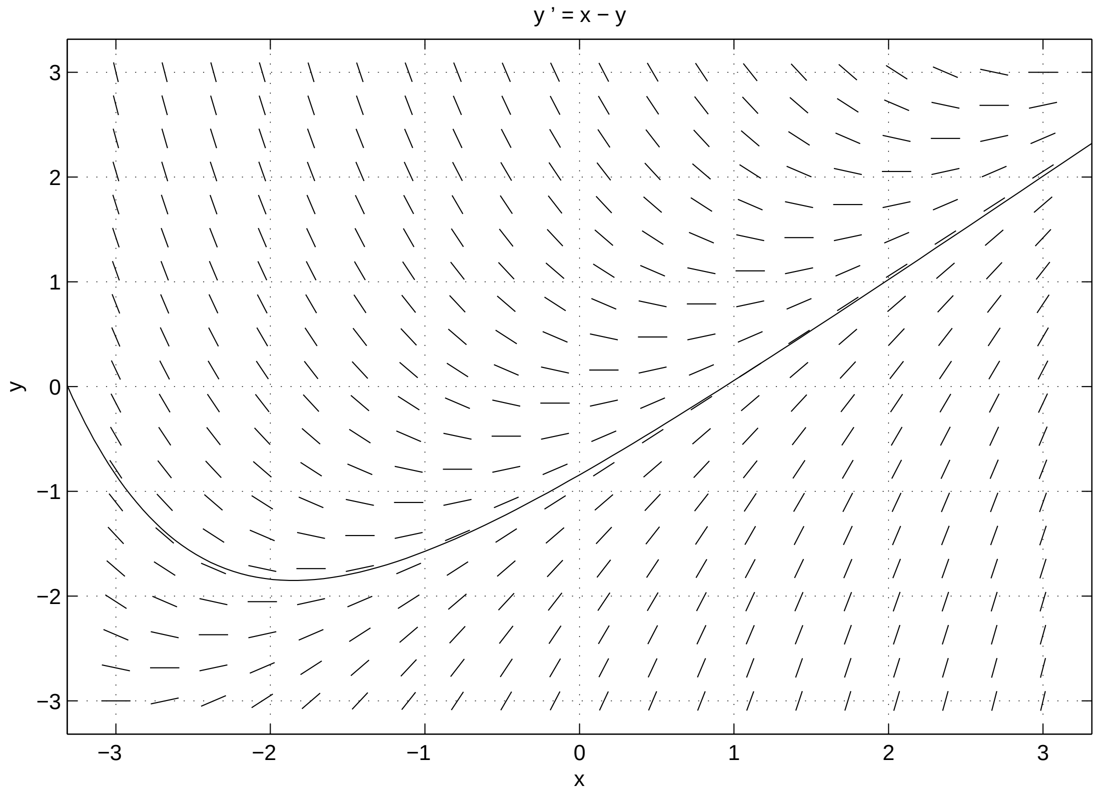
Consider a first order equation in normal form $y' = f(x,y)$. Note that if φ is a solution of this equation with $\varphi(x_0) = x_0$ then the slope of the tangent line to the graph of $y = \varphi(x)$ at $(\overline{x}, \overline{y})$ is given by $f(\overline{x}, \overline{y})$. Since we can compute $f(\overline{x}, \overline{y})$ at every point we can plot the direction a solution will take starting from any given point.

The *Direction Field* for a first order ODE is a figure in which arrows are placed at a grid of points in the xy-plane with an arrow at each point $(\overline{x}, \overline{y})$ of the grid pointing in the direction $f(\overline{x}, \overline{y})$. By starting at a point (x_0, y_0) one can move in the directions of the arrows to get an idea what the solution of the IVP looks like.

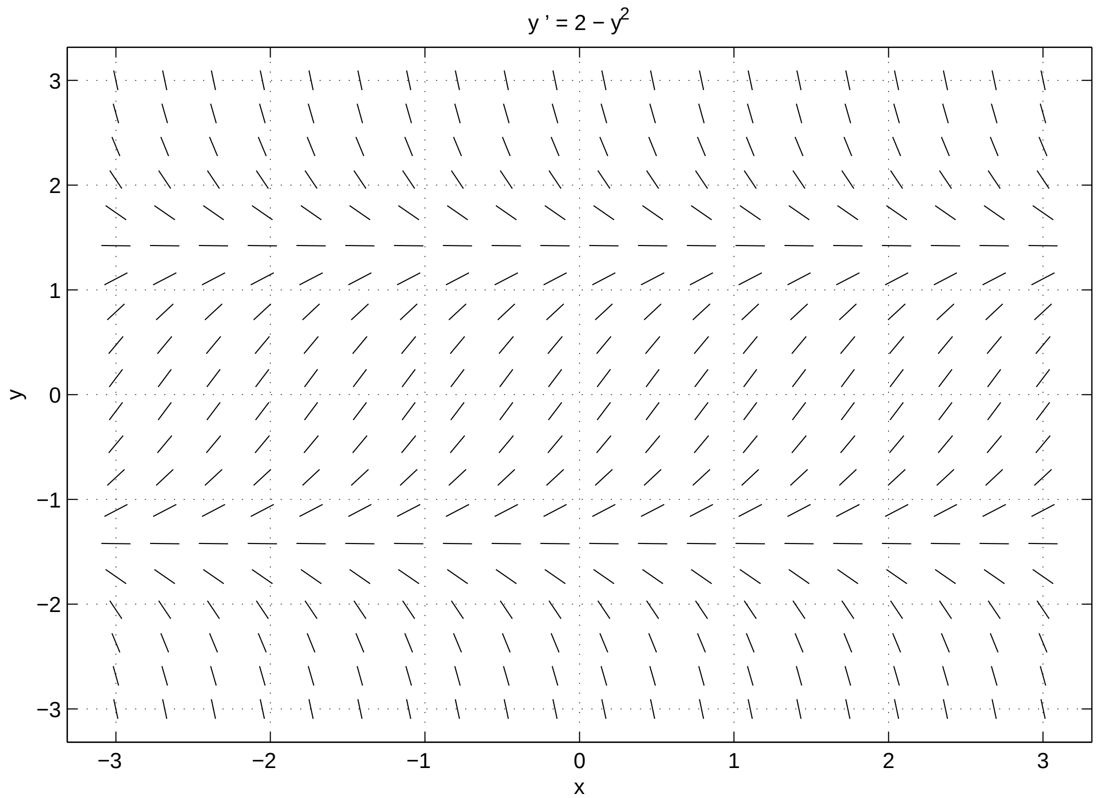



y ' = x + y
3
2
1
0
−1
−2
−3
y
−3
−2
−1
0
1
2
3
x

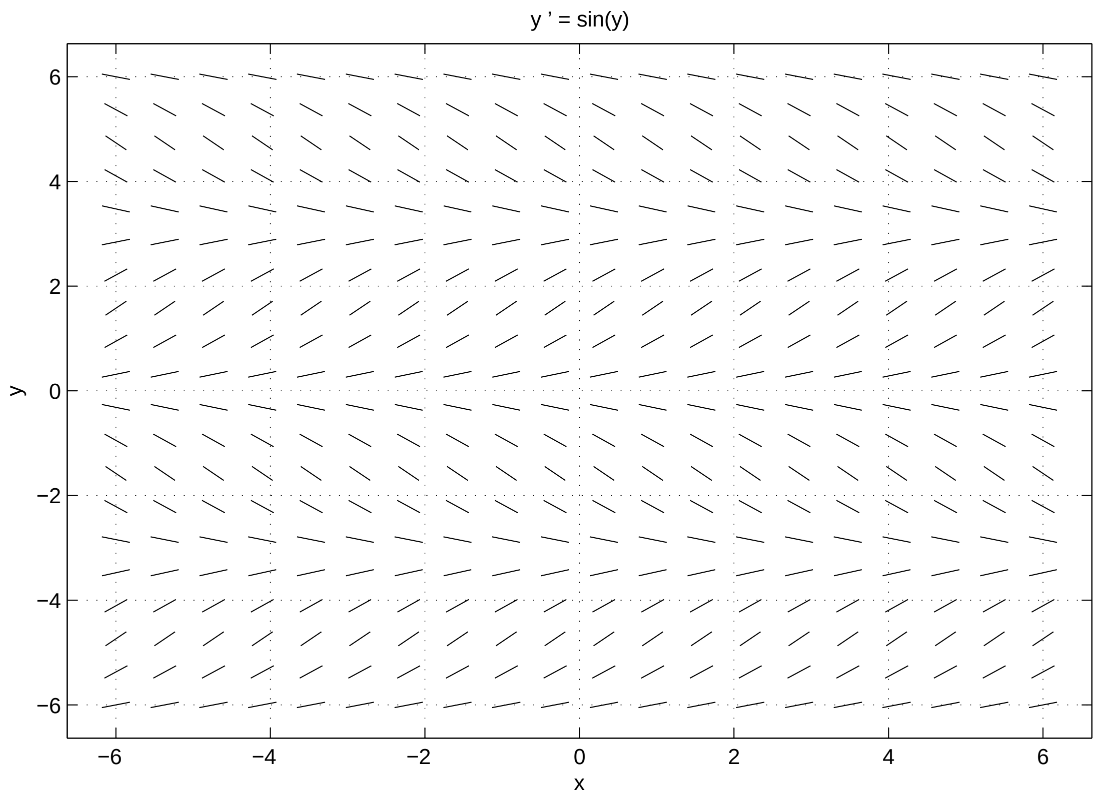

y ' = y − sin(x)
3
2
1
0
−1
−2
−3
y
−3
−2
−1
0
1
2
3
x

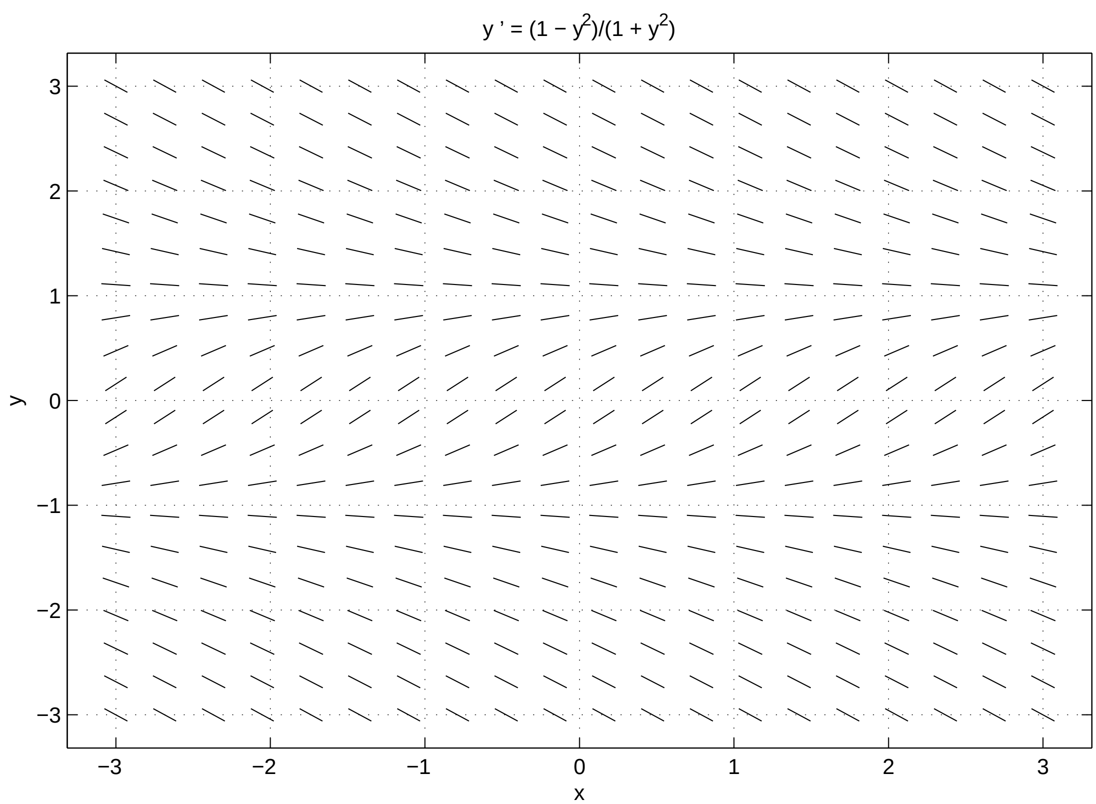


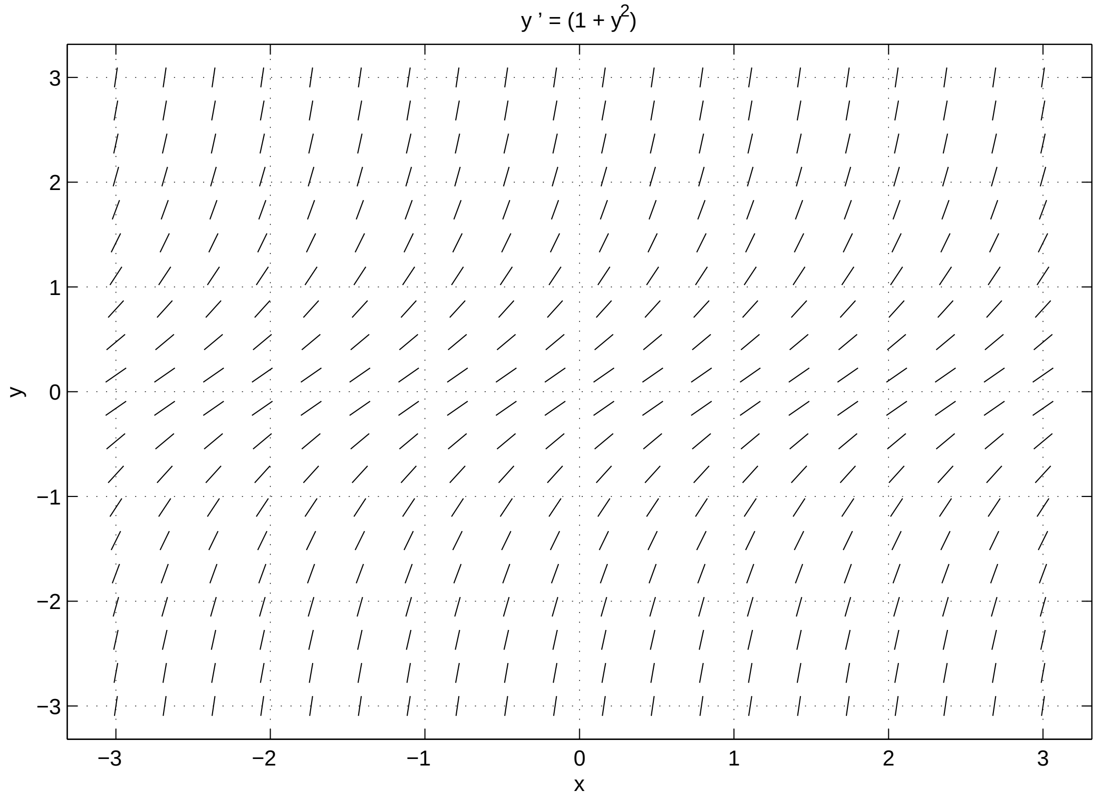
y ' = x − y
3
2
1
0
−1
−2
−3
y
−3
−2
−1
0
1
2
3
x



y ' = 2 − y2
3
2
1
0
−1
−2
−3
y
−3
−2
−1
0
1
2
3
x


y ' = sin(y)
y
x
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6


y ' = (1 − y2)/(1 + y2)
y
x
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3


y ' = (1 + y2)
3
2
1
0
−1
−2
−3
y
−3
−2
−1
0
1
2
3
x