$$\varphi'' = \lambda\varphi,\ \ 0 < x < \ell,\ \varphi(0) = 0,\ \ \varphi(\ell) = 0 \quad \Rightarrow$$
$$\mu_n = \frac{n\pi}{\ell},\quad \lambda_n = -\mu_n^2,\quad \varphi_n(x) = \sqrt{2/\ell}\,\sin(\mu_n x)$$

$$\varphi'' = \lambda\varphi,\ \ 0 < x < \ell,\ \varphi'(0) = 0,\ \ \varphi'(\ell) = 0 \quad \Rightarrow$$
$$\lambda_0 = 0,\ \ \varphi_0 = 1/\sqrt{\ell},\ \ \mu_n = \frac{n\pi}{\ell},$$
$$\lambda_n = -\mu_n^2,\quad \varphi_n(x) = \sqrt{2/\ell}\,\cos(\mu_n x)$$

$$u_t(x,t) = ku_{xx}(x,t),\quad 0 < x < \ell,\quad t > 0$$
$$u(0,t) = 0,\quad u(\ell,t) = 0$$
$$u(x,0) = f(x)$$
$$\mu_n = \frac{n\pi}{\ell},\quad \lambda_n = -\mu_n^2,$$
$$\varphi_n(x) = \sqrt{2/\ell}\,\sin(\mu_n x)$$
$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{k\lambda_n t}\varphi_n(x)$$
$$c_n = \int_0^\ell f(x)\varphi_n(x)\,dx.$$

$$u_t(x,t) = ku_{xx}(x,t),\quad 0 < x < \ell,\quad t > 0$$
$$u_x(0,t) = 0,\quad u_x(\ell,t) = 0$$
$$u(x,0) = f(x)$$
$$\lambda_0 = 0,\quad \varphi_0(x) = 1/\sqrt{\ell},$$
$$\mu_n = \frac{n\pi}{\ell},\quad \lambda_n = -\mu_n^2,$$
$$\varphi_n(x) = \sqrt{2/\ell}\,\cos(\mu_n x)$$
$$u(x,t) = c_0\varphi_0(x) + \sum_{n=1}^{\infty} c_n e^{k\lambda_n t}\varphi_n(x)$$
$$c_n = \int_0^\ell f(x)\varphi_n(x)\,dx.$$

$$u_t(x,t) = ku_{xx}(x,t),\quad 0 < x < \ell,\quad t > 0$$
$$u(0,t) = 0,\quad u_x(\ell,t) = 0$$
$$u(x,0) = f(x)$$
$$\mu_n = \frac{(2n-1)\pi}{2\ell},\quad \lambda_n = -\mu_n^2,\quad \varphi_n(x) = \sqrt{2/\ell}\,\sin(\mu_n x)$$
$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{k\lambda_n t}\varphi_n(x) \quad c_n = \int_0^\ell f(x)\varphi_n(x)\,dx.$$

$$u_t(x,t) = k\big(u_{xx}(x,t) - 2au(x,t)_x + bu(x,t)\big),$$
$$u(0,t) = 0,\quad u(\ell,t) = 0$$

Let $v(x,t) = e^{-(ax+\beta t)}u(x,t),\quad \beta = k(b-a^2).$

$$v_t(x,t) = kv_{xx}(x,t)$$
$$v(0,t) = 0,\quad v(\ell,t) = 0$$
$$v(x,0) = e^{-ax}f(x).$$

$$u_t(x,t) = ku_{xx}(x,t) + R(x),$$
$$u(0,t) = 0,\quad u(\ell,t) = 0$$
$$u(x,0) = f(x)\quad 0 < x < \ell$$

$\Rightarrow$

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{k\lambda_n t}\varphi_n(x) + \sum_{n=1}^{\infty} f_n\left(\frac{e^{k\lambda_n t}-1}{k\lambda_n}\right)\varphi_n(x),$$
$$b_n = \frac{2}{\ell}\int_0^\ell f(x)\varphi_n(x)\,dx,\quad R_n = \frac{2}{\ell}\int_0^\ell R(x)\varphi_n(x)\,dx.$$

Steady State $\psi''(x) = -\frac{1}{k}f(x),\quad \psi(0) = 0,\quad \psi(\ell) = 0,\quad 0 < x < \ell.$

$$
\begin{aligned}
&u_t(x,t) = ku_{xx}(x,t),\\
&u(0,t) = \alpha, \quad u(\ell,t) = \beta\\
&u(x,0) = f(x), \quad 0 < x < \ell,\\
&\text{set} \quad h(x) = \alpha + (\beta - \alpha)\frac{x}{\ell}\\
&v_0(x) = f(x) - h(x)\\
&v(x,t) = u(x,t) - h(x)
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
&v_t(x,t) = kv_{xx}(x,t), \quad 0 < x < \ell, \quad t > 0\\
&v(0,t) = 0, \quad v(\ell,t) = 0\\
&v(x,0) = v_0(x)\\
&v(x,t) = \sum_{n=1}^{\infty} b_n e^{k\lambda_n t}\varphi_n(x), \quad b_n = \frac{2}{\ell}\int_0^{\ell} v_0(x)\varphi_n(x)\,dx\\
&u(x,t) = h(x) + \sum_{n=1}^{\infty} b_n e^{k\lambda_n t}\varphi_n(x)
\end{aligned}
$$

$$
\begin{aligned}
&u^{(2)}_{xx}(x,y) + u^{(2)}_{yy}(x,y) = 0,\\
&(x,y) \in [0,a] \times [0,b],\\
&u^{(2)}(0,y) = 0, \quad u^{(2)}(a,y) = 0\\
&u^{(2)}(x,0) =, \quad u^{(2)}(x,b) = f_1(x)
\end{aligned}
$$

$$
\begin{aligned}
&\mu_n = \left(\frac{n\pi}{a}\right), \quad \lambda_n = \mu_n^2, \quad \varphi_n(x) = \sin(\mu_n x), \quad n = 1, 2, \cdots,\\
&u^{(2)}(x,y) = \sum_{n=1}^{\infty} b_n \sin(\mu_n x)\sinh(\mu_n y)\\
&b_n = \frac{2}{a\sinh(\mu_n b)}\int_0^a f_1(x)\sin(\mu_n x)\ dx.
\end{aligned}
$$

$$
\begin{aligned}
&u^{(3)}_{xx}(x,y) + u^{(3)}_{yy}(x,y) = 0,\\
&(x,y) \in [0,a] \times [0,b],\\
&u^{(3)}(0,y) = 0, \quad u^{(3)}(a,y) = g_1(y)\\
&u^{(3)}(x,0) = 0, \quad u^{(3)}(x,b) = 0
\end{aligned}
$$

$$
\begin{aligned}
&\mu_n = \left(\frac{n\pi}{b}\right), \quad \lambda_n = \mu_n^2, \quad \varphi_n(x) = \sin(\mu_n y), \quad n = 1, 2, \cdots\\
&u^{(3)}(x,y) = \sum_{n=1}^{\infty} b_n \sin(\mu_n y)\sinh(\mu_n x)\\
&b_n = \frac{2}{b\sinh(\mu_n a)}\int_0^b g_1(y)\sin(\mu_n y)\ dy.
\end{aligned}
$$

$$
\begin{aligned}
&u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0, \quad 0 < r < R, \ -\pi \le \theta \le \pi\\
&u(R,\theta) = f(\theta), \quad -\pi \le \theta \le \pi\\
&u(r,\theta) \text{ bounded.}\\
&u(r,\theta) = a_0 + \sum_{m=1}^{\infty}\left(\frac{r^m}{R^m}\right)[a_m\cos(m\theta) + b_m\sin(m\theta)]\\
&a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)\,d\theta\\
&a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\cos(m\theta)\,d\theta\\
&b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\sin(m\theta)\,d\theta
\end{aligned}
$$