Formulas Cullen Zill AEM Chapters 1 and 2

I. (RHS only contains x) $\boxed{y' = f(x)}$ $\Rightarrow$ $\boxed{y = \int f(x)\,dx}$

II. (Separable) $\boxed{f(y)y' = g(x)}$ $\Rightarrow$ $\boxed{\int f(y)\,dy - \int g(x)\,dx = C}$

Implicit and explicit:

1. If we can write the answer as $y = \varphi(x)$ then we have an explicit answer.
2. If we leave the answer in the form $F(x, y) = C$ we have an implicit solution.

III. (First Order Linear) $\boxed{y' + P(x)y = Q(x)}$ Integrating Factor $\boxed{\mu = e^{\int P dx}}$ and the General Solution is $\boxed{y = 1/\mu(x)\left[\int^x \mu(t)Q(t)\,dt + C\right]}$

IV. (Exact) $\boxed{M(x, y)\,dx + N(x, y)\,dy = 0}$ is exact if $\boxed{\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}}$. If exact then there exists $F(x, y)$ so that $\boxed{M = \dfrac{\partial F}{\partial x}, \quad N = \dfrac{\partial F}{\partial y}}$. Use these to find solution $\boxed{F(x, y) = C}$.

V. (Substitution: RHS Linear in x and y) $\boxed{y' = f(ax + by + c)}$ transforms the problem to $\boxed{v = ax + by + c \quad \Rightarrow \quad v' = a + bf(v)}$ which is separable.

VI. (Substitution: RHS is a function of y/x) $\boxed{y' = f(y/x)}$ $\boxed{v = y/x \quad \Rightarrow \quad xv' + v = f(v)}$ is separable.

VII. (Substitution: Bernoulli) $\boxed{y' + P(x)y = Q(x)y^n, n \neq 0, 1}$ then the substitution $\boxed{v = y^{1-n}}$ provides a first oder linear equation for v $\boxed{v' + (1 - n)P(x)v = (1 - n)Q(x)}$ which is First Order Linear.

VIII. (Euler's Numerical Method) For the initial value problem $\boxed{y' = f(x, y), \quad y(x_0) = y_0}$ set $x_n = x_0 + nh,\ n = 0, 1, \cdots$ (small h) and set $y_n \approx y(x_n)$ given by $\boxed{y_{n+1} = y_n + hf(x_n, y_n)}$.