Formulas for Integrals

1. $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$
2. $\displaystyle\int \cos(x)\,dx = \sin(x)$
3. $\displaystyle\int \sin(x)\,dx = -\cos(x)$
4. $\displaystyle\int \sec^2(x)\,dx = \tan(x)$
5. $\displaystyle\int \csc^2(x)\,dx = -\cot(x)$
6. $\displaystyle\int \sec(x)\tan(x)\,dx = \sec(x)$
7. $\displaystyle\int \csc(x)\cot(x)\,dx = -\csc(x)$
8. $\displaystyle\int e^{cx}\,dx = \frac{e^{cx}}{c}$
9. $\displaystyle\int \frac{dx}{x} = \ln(x)$
10. $\displaystyle\int a^x\,dx = \ln(a)a^x$
11. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x)$
12. $\displaystyle\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x)$
13. $\displaystyle\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}(x)$
14. $\sin^2(x) + \cos^2(x) = 1$
15. $\sin^2(x) = (1-\cos(2x))/2$
16. $\cos^2(x) = (1+\cos(2x))/2$
17. $\sin(x)\cos(x) = (1/2)\sin(2x)$
18. $\displaystyle\sinh(x) = \frac{e^x - e^{-x}}{2}$
19. $\displaystyle\cosh(x) = \frac{e^x + e^{-x}}{2}$
20. $\displaystyle\int \sinh(x)\,dx = \cosh(x)$
21. $\displaystyle\int \cosh(x)\,dx = \sinh(x)$
22. $\displaystyle\int \tan(x)\,dx = \ln(|\sec(x)|)$
23. $\displaystyle\int \sec(x)\,dx = \ln(|\sec(x)+\tan(x)|)$
24. $\displaystyle\int \csc(x)\,dx = -\ln(|\csc(x)+\cot(x)|)$
25. $\displaystyle\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$
26. $\displaystyle\int f'g = fg - \int fg'$

27 a. $\displaystyle\int x\sin(x)\,dx = \int x(-\cos(x))'\,dx = -x\cos(x) - \int x'(-\cos(x))\,dx = -x\cos(x) + \sin(x)$

27 b. $\displaystyle\int x\ln(x)\,dx = \int \left(\frac{x^2}{2}\right)'\ln(x)\,dx = \frac{x^2}{2}\ln(x) - \int \frac{x^2}{2}\frac{1}{x}\,dx = \frac{x^2}{2}\ln(x) - \frac{x^2}{4}$

27 c.

$$\begin{aligned}\int e^{ax}\cos(nx)\,dx &= \int e^{ax}\left(\frac{\sin(nx)}{n}\right)'\,dx = e^{ax}\frac{\sin(nx)}{n} - \frac{a}{n}\int e^{ax}\sin(nx)\,dx \\ &= e^{ax}\frac{\sin(nx)}{n} - \frac{a}{n}\int ae^{ax}\left(-\frac{\cos(nx)}{n}\right)'\,dx \\ &= e^{ax}\frac{\sin(nx)}{n} + \frac{a}{n^2}e^{ax}\cos(nx) - \frac{a^2}{n^2}\int e^{ax}\cos(nx)\,dx\end{aligned}$$

so $\displaystyle\int e^{ax}\cos(nx)\,dx = \frac{e^{ax}}{(n^2+a^2)}(n\sin(nx) + a\cos(ax))$.