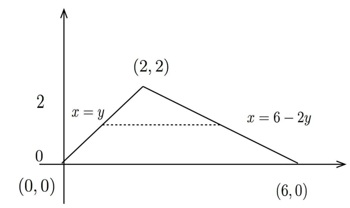
Math 2350 Fall 2007, Sample Test # 3, Name ______________________

1. Evaluate $\int_0^4 \int_{-1}^1 (2x+y)\, dxdy$

ANSWER: $\boxed{\int_0^4 [x^2+xy]\big|_{-1}^1 \, dy = \int_0^4 2y\, dy = 16}$

2. Express $\iint_D f(x,y)\, dA$ as an iterated integral where D is the triangular region with vertices $(0,0)$, $(2,2)$ and $(6,0)$.

ANSWER: $\boxed{\int_0^2 \int_y^{6-2y} f(x,y)\, dxdy}$



3. Evaluate the integral $\int_0^2 \int_y^2 \cos(x^2)\, dxdy$ by reversing the order of integration. Give both, the new iterated integral and the final answer.

ANSWER: $\boxed{\int_0^2 \int_0^x \cos(x^2)\, dydx = \int_0^2 x\cos(x^2)\, dx = \frac{1}{2}\int_0^4 \, du = \frac{\sin(4)}{2}}$

4. Set up integrals in **both** rectangular and polar form for the area of the surface of the paraboloid $z = 32 - (2x^2 + 2y^2)$ that lies in the first octant.

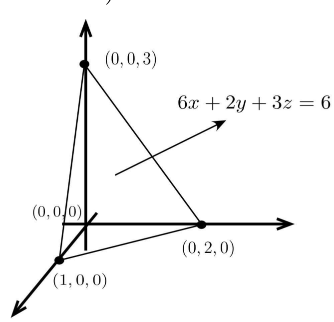
ANSWER: $\boxed{\int_0^4 \int_0^{\sqrt{16-x^2}} \sqrt{1+16x^2+16y^2}\, dydx = \int_0^{\pi/2} \int_0^4 \sqrt{1+16r^2}\, rdrd\theta}$

5. Set up a triple integral $\iiint_R x\, dV$ where R is the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$ and $(0,0,3)$. Express dV appropriately and determine the corresponding six limits of integration needed if one were to evaluate the integral (BUT DO NOT EVALUATE) .

ANSWER:

Plane through the points: $6x + 2y + 3z = 6$,

$$\int_0^1 \int_0^{2-2x} \int_0^{(6-6x-2z)/3} x\, dzdydx$$



6. Evaluate the triple integral $\int_0^1 \int_0^{(1-x)} \int_{-\sqrt{x}}^{\sqrt{x}} 15\, dydzdx.$

ANSWER: $\boxed{8}$

7. Show that the vector field $\boldsymbol{F}(x,y,z) = \langle z\sin(y), xz\cos(y), x\sin(y)\rangle$ is conservative.

ANSWER:

$$\nabla\times\boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \partial_x & \partial_y & \partial_z \\ z\sin(y) & xz\cos(y) & x\sin(y) \end{vmatrix}$$

$$= \langle (x\cos(y) - x\cos(y)), (\sin(y) - \sin(y)), (z\cos(y) - z\cos(y))\rangle = \mathbf{0}$$

8. The vector field $\boldsymbol{F} = \langle 2xe^{3y}, (2+3x^2e^{3y})\rangle$ is conservative. Find a potential function, i.e. find f so that $\nabla f = \boldsymbol{F}$.

ANSWER:

$f_x = 2xe^{3y},\ \ f_y = (2+3x^2e^{3y}) \ \Rightarrow\ f = x^2e^{3y} + h(y)$

so $(2+3x^2e^{3y}) = f_y = x^2 3e^{3y} + h'(y)$ and $h'(y) = 2$

Therefore

$$f(x,y) = x^2e^{3y} + 2y$$

9. Evaluate the line integral $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$ where $\boldsymbol{F} = \langle y, -x, z\rangle$ and $\boldsymbol{r}(t)$ is the line segment joining the point $(1,0,-2)$ to the point $(2,4,1)$.

ANSWER:

The line from $(1,0,-2)$ to $(2,4,1)$ can be written parametrically as $x = t+1$, $y = 4t$, $z = 3t-2$ for $0 \le t \le 1$. Thus $dx = dt$, $dy = 4dt$, $dz = 3dt$ and we have

$$\int_C [y\,dx - x\,dy + z\,dz] = \int_0^1 [(4t)(dt) - (t+1)(4\,dt) + (3t-2)(3\,dt)]$$

$$= \int_0^1 [(4t) - 4(t+1) + 3(3t-2)]\,dt$$

$$= \int_0^1 [9t - 10]\,dt = \left[9\frac{t^2}{2} - 10t\right]\Big|_0^1 = \frac{9}{2} - 10 = -\frac{11}{2}$$