Math 2350 Fall 2007, Sample Test # 2, Name ________________________

1. Use polar coordinates to find the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sin(2(x^2+y^2))}$.

ANSWER: $\boxed{1/2}$

2. Find the limit or show it does not exist $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2}{(2x^2+y^2)}$.

ANSWER: $\boxed{x=0, y\to 0 \Rightarrow 0,\ x\to 0, y=0 \Rightarrow 3/2 \text{ Therefore the limit DNE}}$

3. Use implicit differentiation to find z_x and z_y given $\ln(xy+yz+z)=5$

ANSWER: $\boxed{z_x = -\frac{y}{y+1}, \quad z_y = -\frac{x+z}{y+1}}$

4. Find the directional derivative of $z=f(x,y)=\ln(3x+y^2)$ at $P_0(0,1)$ in the direction $\boldsymbol{v}=\boldsymbol{i}-\boldsymbol{j}$.

ANSWER: $\boxed{D_{\boldsymbol{u}}f(P_0) = \langle 3,2\rangle \cdot \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle = \frac{1}{\sqrt{2}}}$

5. Find $\dfrac{\partial F}{\partial t}$ if $F(x,y,z)=x^2y+y^2z+xz$, and $x=2t$, $y=t^2$ and $z=\sin(t)$.

ANSWER: $\boxed{\frac{dF}{dt} = (2xy+z)(2)+(x^2+2yz)(2t)+(y^2+x)(\cos(t))}$

6. Find the *direction*, $\boldsymbol{N}$, and *magnitude* of most rapid increase of the function $z=F(x,y)=xe^{x-y}$ at the point $P_0(1,1,1)$.

ANSWER: $\boxed{\nabla F(P_0) = \langle 2,-1\rangle \Rightarrow \boldsymbol{N} = \frac{\langle 2,-1\rangle}{\sqrt{5}}, \quad \|\nabla F(P_0)\| = \sqrt{5}}$

7. Find a unit normal vector $\boldsymbol{N}$ and a standard form equation of the tangent plane to the level surface $F(x,y,z)=x^2-y^3+z=3$ at $P_0(-1,-1,1)$.

ANSWER: $\boxed{\nabla F = \langle 2x, -3y^2, 1\rangle, \boldsymbol{N} = \frac{\langle -2,-3,1\rangle}{\sqrt{14}}, \quad -2x-3y+z-6=0}$

8. Find the critical points of $z=f(x,y)=x^5+y^4-5x-32y$ and classify (as rel. Max., rel. min. or saddle point).

ANSWER: $\boxed{(1,2) \text{ Rel. Min },\quad (-1,2) \text{ Saddle}}$

9. Find the absolute maximum and absolute minimum of $z=f(x,y)=x^2+y^2-3y$ on $x^2+y^2\le 4$.

ANSWER: $\boxed{\text{Abs. Min. } f(0,3/2) = -9/4, \quad \text{Abs. Max. } f(0,-2)=10}$