Math 2350 Fall 2007, Sample Test # 1, Name _____

- 1. Given $\mathbf{F} = 2\sin(t)\mathbf{i} t\mathbf{j} + t\mathbf{k}$ and $\mathbf{G} = t\mathbf{i} \frac{2}{t}\mathbf{j} + t\mathbf{k}$, $\mathbf{H} = \mathbf{i} t\mathbf{k}$. Find $\mathbf{F} \cdot (\mathbf{G} \times \mathbf{H})$. ANSWER: $2 + 4\sin(t) - t(t + t^2)$ 2. Find the limit $\lim_{t \to 0} \left[\frac{e^{t+1}}{e^{t-1}}\mathbf{i} + \frac{\sin(2t)}{\sin(3t)}\mathbf{j} + t\ln(t)\mathbf{k}\right]$. ANSWER: $e^2\mathbf{i} + \frac{2}{3}\mathbf{j}$
- 3. Given $\mathbf{F} = \mathbf{i} + e^t \mathbf{j} + t \mathbf{k}$ and $\mathbf{G} = t \mathbf{i} t^2 \mathbf{j}$, find $\frac{d}{dt} (\mathbf{F} \times \mathbf{G})(t)$ ANSWER: $3t^2 \mathbf{i} + 2t \mathbf{j} - (2t + te^t + e^t)\mathbf{k}$
- 4. Given the velocity vector $\mathbf{V}(t) = -2t\mathbf{i} + 4(\sin^2(t))\mathbf{j} + 2e^{2t}\mathbf{k}$ and the initial position vector $\mathbf{R}(0) = \mathbf{i} \mathbf{k}$, find the position vector $\mathbf{R}(t)$.

ANSWER:
$$\mathbf{R}(t) = (-t^2 + 1)\mathbf{i} + (2t - \sin(2t))\mathbf{j} + (e^{2t} - 2)\mathbf{k}$$

5. Find the time of flight T_f and range R_f for a projectile fired (in a vacuum) from ground level at the angle $\alpha = 45^{\circ}$ with initial speed $v_0 = 80 ft/s$. Assume that $g = 32 ft/s^2$.

ANSWER:
$$T_F = \frac{2 \cdot 80 \cdot \sin(45^\circ)}{32} = \frac{5\sqrt{2}}{2}, \quad R_F = \frac{80^2 \sin(90^\circ)}{32} = 200$$

- 6. Find the length of the curve $\mathbf{R}(t) = 4\cos(t)\mathbf{i} 4\sin(t)\mathbf{j} + 5t\mathbf{k}$ from t = 0 to t = 2. ANSWER: $2\sqrt{41}$
- 7. Find the curvature of $y = x + x^{-1}$ at x = 1.

ANSWER: 2

8. Given $\mathbf{R}(t) = (2-t)\mathbf{i} + (1+4t)\mathbf{j} - 8t\mathbf{k}$, Find an expression for \mathbf{R} in terms of arc length s measure from where t = 0 in the direction of increasing t.

ANSWER: $\mathbf{R} = (2 - s/9)\mathbf{i} + (1 + 4s/9)\mathbf{j} - 8s/9\mathbf{k}$

9. Given $\mathbf{R}(t) = (2-t)\mathbf{i} + (1+4t)\mathbf{j} - 8t\mathbf{k}$ (see problem 8), find the Unit Tangent Vector in terms of either t or s.

ANSWER: $T = \frac{-i + 4j - 8k}{9}$