Lines in $\mathbb{R}^3$

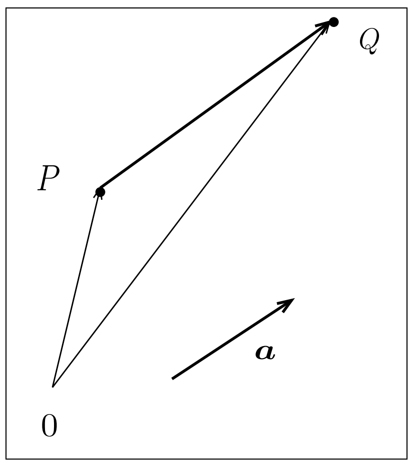
Parametric form: A line L in $\mathbb{R}^3$ passing through a point P (x_0, y_0, z_0) and parallel to a vector $\boldsymbol{a} = A\boldsymbol{i} + B\boldsymbol{j} + C\boldsymbol{k}$ has parametric equation

$$x = x_0 + sA, \quad y = y_0 + sB, \quad z = z_0 + sC \quad s \in \mathbb{R}$$

where Q (x, y, z) is a point on the line.

This follows since the vectors $\langle (x - x_0), (y - y_0), (z - z_0) \rangle$ and $\boldsymbol{a}$ must be parallel. Recall that two vectors are parallel if and only if they are a multiple of each other. So we must have

$$\langle (x - x_0), (y - y_0), (z - z_0) \rangle = s\boldsymbol{a} = t\langle A, B, C \rangle \quad \text{for some} \quad s \in \mathbb{R}.$$



Another form of the equation of the line is obtained by solving for the parameter s. This provides the so-called *Symmetric Form of a line in* $\mathbb{R}^3$:

$$\frac{(x - x_0)}{A} = \frac{(y - y_0)}{B} = \frac{(z - z_0)}{C}.$$

A *Tangent Line* to a curve $\boldsymbol{F}(t) = \langle f_1(t), f_2(t), f_3(t) \rangle$ in $\mathbb{R}^3$ can easily be obtained by noting that the vector derivative $\boldsymbol{F}'(t) = \langle f_1'(t), f_2'(t), f_3'(t) \rangle$ is a tangent vector to the curve for each t. Therefore, at a fixed point t_0, the tangent line can be expressed as

$$\langle (x - f_1(t_0)), (y - f_2(t_0)), (z - f_3(t_0)) \rangle = s\langle f_1'(t_0), f_2'(t_0), f_3'(t_0) \rangle \text{ for some} \quad s \in \mathbb{R}.$$

This can also be wriiten as

$$x = f_1(t_0) + sf_1'(t_0), \quad y = f_2(t_0) + sf_2'(t_0), \quad z = f_3(t_0) + sf_3'(t_0).$$