Formulas for Arc Length and Curvature

In these notes we use notation like $\boldsymbol{R}$ for a vector valued function

$$\boldsymbol{R}(t) = \langle f_1(t), f_2(t), f_3(t)\rangle = f_1(t)\boldsymbol{i} + f_2(t)\boldsymbol{j} + f_3(t)\boldsymbol{k}$$

which generates a curve C (in $\mathbb{R}^2$ or $\mathbb{R}^3$). We recall that $\boldsymbol{V}(t) = \boldsymbol{R}'(t)$ is a tangent vector and $\boldsymbol{A}(t) = \boldsymbol{V}'(t) = \boldsymbol{R}''(t)$ is the acceleration vector.

Unit Tangent Vector $\boldsymbol{T}(t) = \dfrac{\boldsymbol{R}'(t)}{\|\boldsymbol{R}'(t)\|}$, (or in arc length) $\boldsymbol{T} = \dfrac{d\boldsymbol{R}}{ds}(s)$

Principal Unit Normal Vector $\boldsymbol{N}(t) = \dfrac{\boldsymbol{T}'(t)}{\|\boldsymbol{T}'(t)\|}$ (or in arc length) $\boldsymbol{N} = \dfrac{1}{\kappa}\dfrac{d\boldsymbol{T}}{ds}(s)$

Arc Length	$s(t)$	$\int_0^t \|\boldsymbol{R}'(u)\|\, du$
Curvature	κ	$\left\| \dfrac{d\boldsymbol{T}}{ds} \right\|$
Curvature (two deriv form)	κ	$\dfrac{\|\boldsymbol{T}'(t)\|}{\|\boldsymbol{R}'(t)\|}$
Curvature (Cross deriv form)	κ	$\dfrac{\|\boldsymbol{R}'(t) \times \boldsymbol{R}''(t)\|}{\|\boldsymbol{R}'(t)\|^3}$
Curvature (functional form)	$y = f(x)$	$\dfrac{\|f''(x)\|}{(1 + [f'(x)]^2)^{3/2}}$