

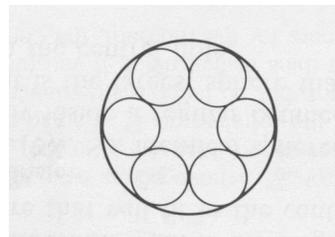
EMMY NOETHER MIDDLE SCHOOL MATHEMATICS DAY
Texas Tech University
May 14, 2014

Write your name, the name of your school and your current grade level on the front of the blue book. Work all problems. Show your reasoning and clearly indicate your answer to each problem. Do not simply claim an answer. Partial credit may be given where appropriate. Each problem is worth 10 points. If you are not sure how to approach a problem, you are strongly encouraged to experiment and to try to discover.

1.) The whole number 12 is exactly 4 times the sum of its digits ($12 = 4(1+2)$). Can you determine a whole number that is exactly three times the sum of its digits? Is such a number unique, i.e. is there only one such number? Either determine all such numbers, show that there is only one such number or show that no such number exists.

2.) How many four letter words can be formed from the letters T E X A S T E C H? A word is any four of the above letters in a specified order, e.g. SXAT. A word does not have “meaning” or be pronounceable in any particular language.

3.) Six identical circles are tightly arranged around the inside of a larger circle. The larger circle has radius 1. What is the radius of the largest circle that will fit in the central hole?



4.) On a given day the Sun, Earth and Mars are in a line with Earth and Mars on the same side of the Sun. Assume that each planet goes around the Sun in the same direction and that each planet moves in a circle around the Sun at a constant rate. The Earth goes around the Sun in 365 days and Mars goes around the Sun in 687 days. How long will it be until the Sun, Earth and Mars are again in a line with Earth and Mars on the same side of the Sun?

5.) A digital clock displays the hours and minutes. How many times between 12:00 noon and 12:00 midnight will the clock display a time with at least two digits the same, e. g. 1:21? Count 12:00 noon as such a time but do not count 12:00 midnight as another such time, since it is the same display on the clock. In a time such as 1:20 the clock does not display an initial 0, i.e. it does not display 01:20 and so this is not a time with two digits the same.

6. Mathematician Mary Ellen Rudin was born on December 7, 1924. This year, December 7, 2014 (90 years later) occurs on a Sunday. On what day of the week was Mary Ellen Rudin born? Indicate your reasoning and do not simply state (or guess) a day of the week. Remember to allow for leap years.