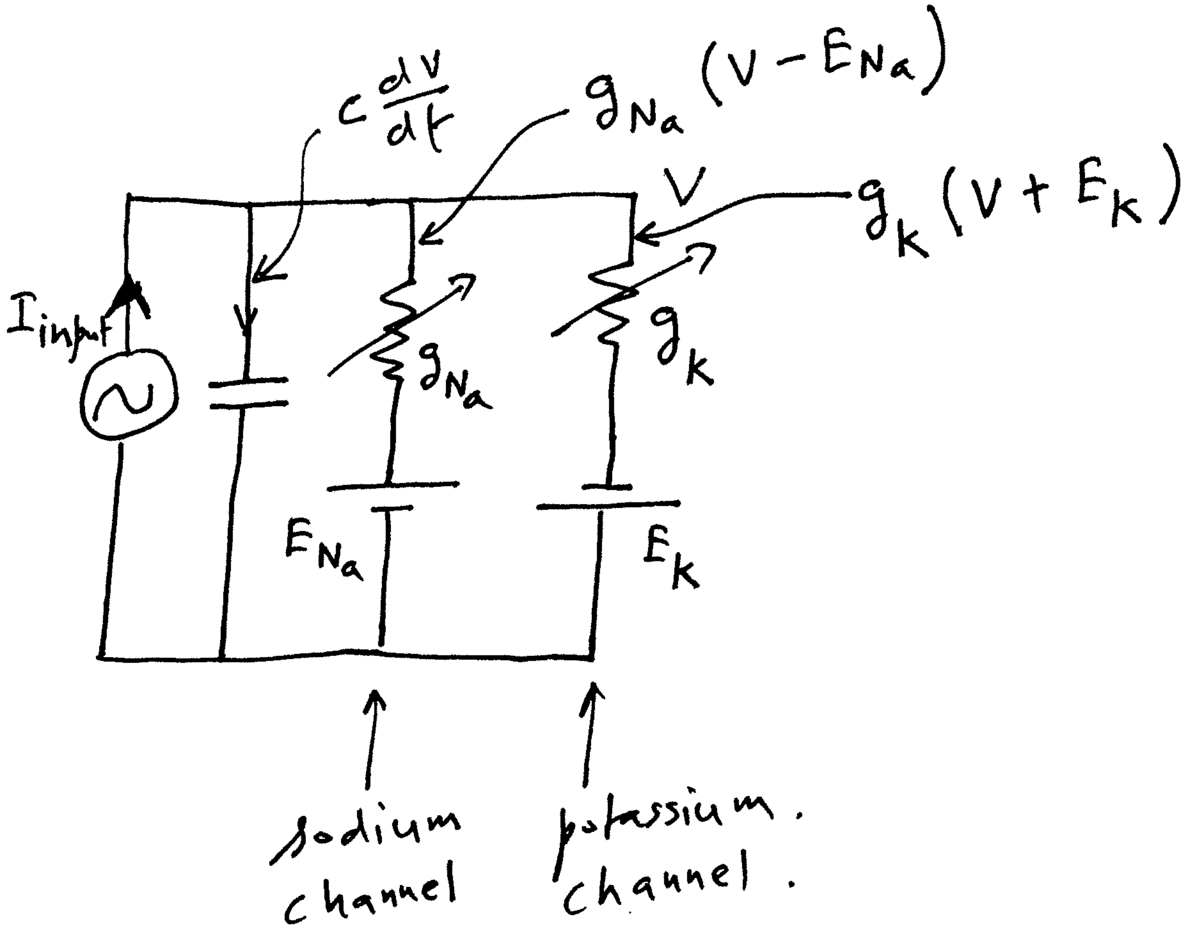
# Lec 2

The goal of this lecture is to elaborate Exercise 1·6 in sufficient details.




The point is that $g_{Na}$ and $g_K$ are not simple functions but, as was pointed out by Hodgkin-Huxley in 1952, sufficiently complicated.

## Exercise 2·1 (HW 2)

Redo exercise 1·6, (and this time correctly).

Instead of choosing $g_{Na}$ and $g_K$ to be p.w. constant let us consider the following.

$$g_{Na} = \bar{g}_{Na} m^3 h \qquad (2\cdot1)$$

$$g_K = \bar{g}_K n^4 \qquad (2\cdot2)$$

where $\bar{g}_{Na} = 20$; $\bar{g}_K = 8$.

$m, h, n$ are dynamic variables given by

$$\begin{aligned}
\dot{m} &= \alpha_m (1-m) - \beta_m m \\
\dot{h} &= \alpha_h (1-h) - \beta_h h \qquad (2\cdot3) \\
\dot{n} &= \alpha_n (1-n) - \beta_n n
\end{aligned}$$

$$\alpha_n(V) = \frac{\cdot 01(10-V)}{e^{\frac{10-V}{10}} - 1}; \quad \beta_n(V) = \cdot 125\, e^{-V/80} \qquad (2\cdot4)$$

$$\alpha_m(V) = \frac{.1(25-V)}{e^{\frac{25-V}{10}} - 1} \; ; \; \beta_m(V) = 4e^{-V/18} \qquad (2.5)$$

$$\alpha_h(V) = .07e^{-V/20} \; ; \; \beta_h(V) = \frac{1}{e^{\frac{(30-V)}{10}} + 1} \qquad (2.6)$$

Consider the membrane equation

$$\dot{V} = -g_{Na}(V-50) - g_k(V+90) + I_{input} \qquad (2.7)$$

which is same as (1.15) except that 4 has been absorbed in $\bar{g}_{Na}$ and $\bar{g}_k$.

(a): Assuming V to be constant calculate $g_{Na}$ and $g_k$ in the steady state as a function of V. Please plot the two functions.

(b) Assume $m(0) = n(0) = h(0) = 0$ and assume $V(0)$ to be same as Exercise 1.6. Solve for $V(t)$ using matlab and plot. Choose $I_{input}$ in the range $0 < I_{input} < 10$.

## Exercise 2·2 (H.W.2)

In this exercise, we will do what Rinzel did back in 1985. He wrote (2·3) as.

$$\dot{m} = \frac{1}{\tau_m(V)}\left(-m + M(V)\right)$$

$$\dot{h} = \frac{1}{\tau_h(V)}\left(-h + H(V)\right) \qquad (2\cdot 8)$$

$$\dot{n} = \frac{1}{\tau_n(V)}\left(-n + N(V)\right)$$

(a) Rinzel claimed that $\tau_m$ is so small for all values of V that the variable m rapidly approaches its equilibrium value M(V). On the basis of Exercise 2·1, do you believe that.

(b) Rinzel claimed that h is close to 1−n. What that meant is the $Na^+$ channel closes at the same rate but in opposite direction to $K^+$ channel opening.

On the basis of exercise 2·1, do you believe that.

ⓒ Irrespective of your answer in ~~(2·1), (2·2)~~ parts ⓐ & ⓑ; substitute $m = m(\infty)$ and $h = 1-n$ in (2·1), (2·2) to obtain

$$g_{Na} = \bar{g}_{Na}\, m^3(\infty)\,(1-n) \qquad (2\cdot 9)$$

$$g_K = \bar{g}_k\, n^4$$

and we have Rinzell's approximation:—

$$\dot{V} = -g_{Na}(V-50) - g_k(V+90) + I_{input}.$$

$$\dot{n} = \alpha_n(1-n) - \beta_n n \qquad (2\cdot 10)$$

Assume $n(0) = 0$, $V(0)$ same as exercise 1·6, solve for $V(t)$ using matlab and plot. Choose $I_{input}$ in the range

$$0 < I_{input} < 10.$$

(d) Is there a threshold current $I_{input}$ for spike generation?

(e) Plot the isocline curve by setting $\dot{V}=0$, i.e.

$$g_{Na}(V-50)+g_K(V+90)=I_{input}$$

choose $I_{input}=0$.

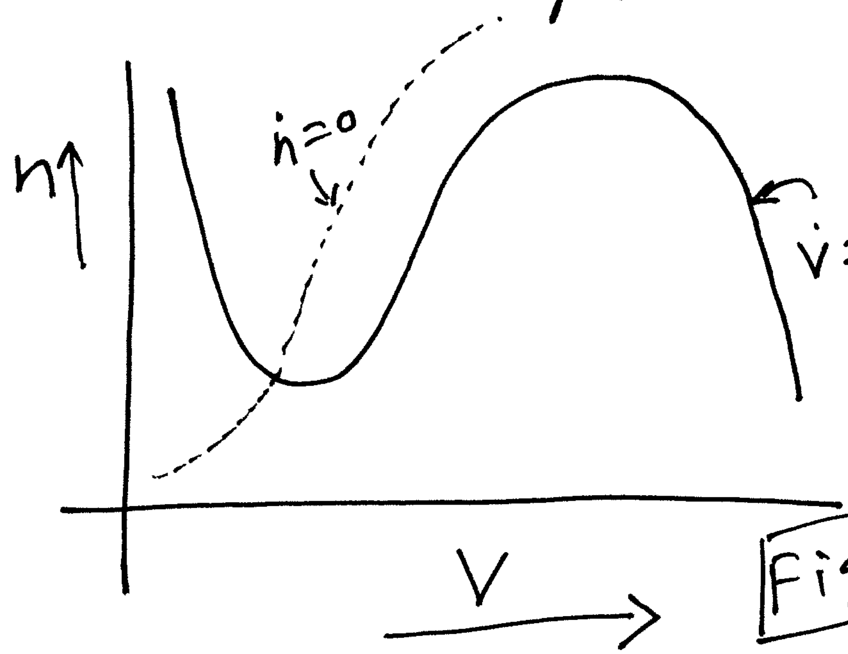



Fig 2.1

Plot the isocline curve by setting $\dot{n}=0$, i.e.

$$\alpha_n(1-n)-\beta_n n=0$$

Remark: The two curves look like the figure above.

## Exercise 2·3 (H.W.2)

FitzHugh-Nagumo Model was derived by FitzHugh (1961) and Nagumo (1962). In this exercise, we will derive this model.

$$\dot{v} = a\left(v - bv^3 - n + I_{input}\right) \qquad (2\cdot 11)$$
$$\dot{n} = c\left(-n + dv + e\right)$$

The isoclines are

$n = v - bv^3 + I_{input}$ for $\dot{v} = 0$

$n = dv + e$ for $\dot{n} = 0$

(a) Find d and e to match the isocline $\dot{n}=0$ in Fig , from exercise 2·2.

(b) Find b to match the isocline $\dot{v}=0$ in Fig , from exercise 2·2.

Choose $a = 10$, $c = \cdot 8$.

(c) Plot the isoclines for the FitzHugh-Nagumo model. They would look like

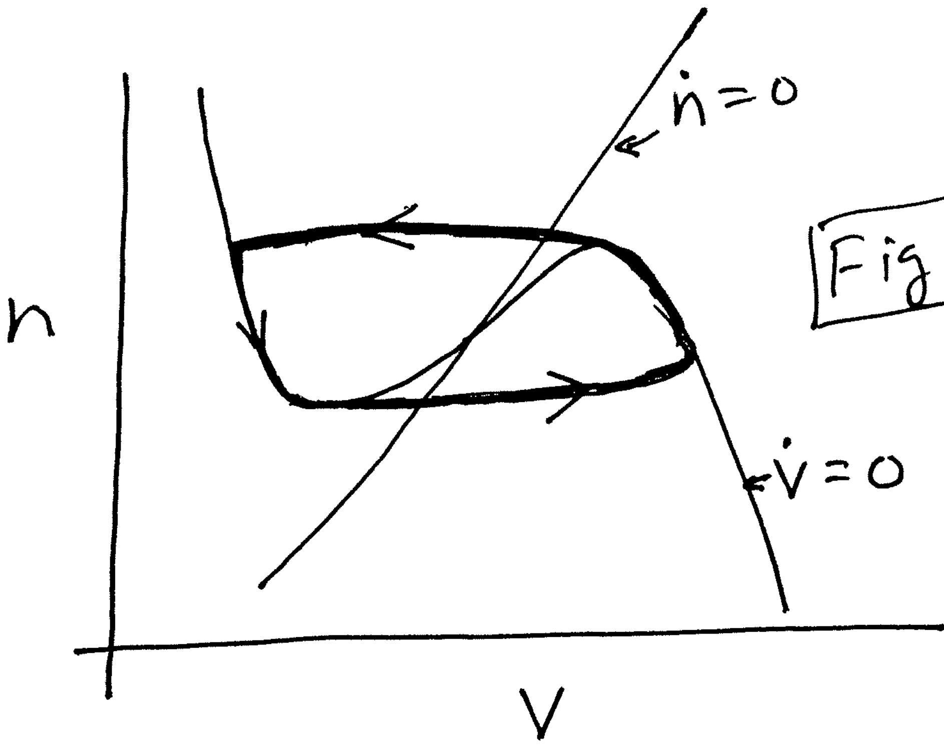



Fig 2·2

(d) Find one value of Iinput which produces action potentials in the FitzHugh Nagumo model. Try Iinput = 1·5. and ·5

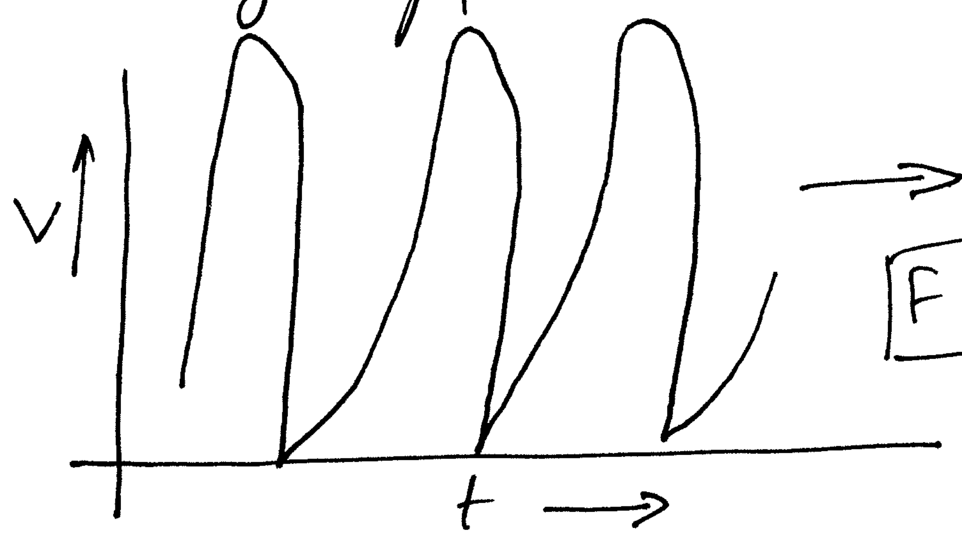



Fig 2·3

In deriving the FitzHugh Nagumo model, we have essentially matched the isoclines but did not keep the channels. Actually the voltage clamp measurements that led to Rinzell's approximation were done much later. We now talk about Wilson's model that also matches isoclines but keeps the channels.

## Exercise 2.4 (H.W. 3)

Rewrite (2.10) as follows:

$$\dot{V} = a\left[-g_{Na}(V)(V-50) - n(V+90) + I_{input}\right]$$

$$\dot{n} = c(-n + G(V)) \qquad (2.12)$$

where we assume that $c = .8$, $a = 10$

$G(V) = \alpha V + \beta$ ← a linear function.

$g_{Na}(V) = 1 + \gamma V + \delta V^2$ ← a quadratic function.

The isocline $\dot{n} = 0$ is given by

$$n = G(V) = \alpha V + \beta .$$

The isocline $\dot{V} = 0$ is given by.

$$n = \frac{-g_{Na}(V)[V-50]}{V+90} \text{, assume } I_{input} = 0.$$

$$\Rightarrow n = \frac{-(1+\gamma V + \delta V^2)(V-50)}{V+90}.$$

(a) Find $\alpha, \beta, \gamma, \delta$ that best fits the isoclines obtained by Rinzell in Exercise 2.

(b) Solve (2·12) with the above values and plot $V(t)$ as a function of $t$. Choose $I_{input}$ in the range $0 \leq I_{input} \leq 10$.

We have an "essential dynamics" proposed by Wilson.