H. W. 6

(1) We are given a dynamical system

$$\dot{\underline{x}} = A\underline{x} + bu$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Calculate a matrix P such that if we set

$$\underline{x} = PZ$$

we have

$$\dot{Z} = \underbrace{\begin{pmatrix} x & 1 & 0 \\ x & 0 & 1 \\ x & 0 & 0 \end{pmatrix}}_{F} Z + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{g} u$$

Also calculate F.

(2) We are given a dynamical system

$$\dot{\underline{x}} = A\underline{x} + bu$$

where

$$A = \begin{pmatrix} -5 & 1 & 0 \\ 4 & 0 & 1 \\ 39 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

We want to calculate

$$F = (f_1 \quad f_2 \quad f_3)$$

such that

$$A + bF$$

has characteristic polynomial given by

$$(\lambda + 1)^3.$$

Can you help us??

(3) We are given a dynamical system

$$\dot{\underline{x}} = A\underline{x} + Bu$$

where A is a 4×4 matrix

B is a 4×2 matrix.

choose

$$A = \begin{pmatrix} 2 & 1 & 3 & 0 \\ 3 & 0 & 4 & 0 \\ 5 & 0 & -1 & 1 \\ 9 & 0 & -7 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

(a) Is this system controllable??

(b) Calculate the Kronecker Indices.

(c) Calculate a 2×4 matrix F such that $A+BF$ has characteristic polynomial at

$$\lambda^4 + 19\lambda^3 + 29\lambda^2 - 15\lambda + 44.$$

We are given a dynamical system

$$\dot{x} = Ax + Bu$$

where

$B = (b_1 \; b_2 \; b_3)$ a 6×3 matrix

A is a 6×6 matrix.

What is known about matrices A & B are the following:

① $b_1, b_2, b_3, Ab_1, Ab_2, A^2 b_1$ are independent vectors of $\mathbb{R}^6$.

② Ab_3 is linearly dependent on vectors

$b_1, b_2, b_3, Ab_1, Ab_2$

Ⅰ Is the system controllable?

Ⅱ What are the Kronecker Indices?

(III) Write out the Kronecker canonical form with 'x' is appropriate places and 0, 1 in other places.

(IV) If

$$Ab_3 = 3b_1 + 2b_2 - 5Ab_2$$

$$A^2b_2 = 4b_2 + 6Ab_2 - 7A^2b_1$$

$$A^3b_1 = -b_3 + Ab_1 + Ab_2 .$$

can you write down the Kronecker canonical form with 'x' replaced by numbers.