H. W. 3

(1) Consider a time varying linear system

$$\dot{x} = A(t)\,x(t) \quad (*)$$

where

$$A(t) = f(t)\,B_1 + g(t)\,B_2$$

where B_1 & B_2 commute. Show that the transition matrix $\phi(t,0)$ of $(*)$ is given by

$$e^{B_1 \int_0^t f(\sigma)\,d\sigma}\; e^{B_2 \int_0^t g(\sigma)\,d\sigma}$$

Hence compute the state transition matrix when

$$A(t) = \begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix}$$

(2) Consider

$$\dot{x}(t) = A(t)\,x(t)$$

where

$$A(t) = \begin{pmatrix} -1+\cos t & 0 \\ 0 & -2+\cos t \end{pmatrix}$$

Floquet-Liapunov tells us that there exists a Liapunov transformation

$$x(t) = P(t)^{-1} z(t)$$

such that

$$\dot{z} = R\,z$$

where R is time invariant. Find $P(t)$ and R.

③ An author claims that there exists a Liapunov Transformation which transforms the constant system.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \sigma & \omega \\ -\omega & \sigma \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Into the constant system.

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

ⓐ Can you show that.

ⓑ Generalize this result to higher dimension.

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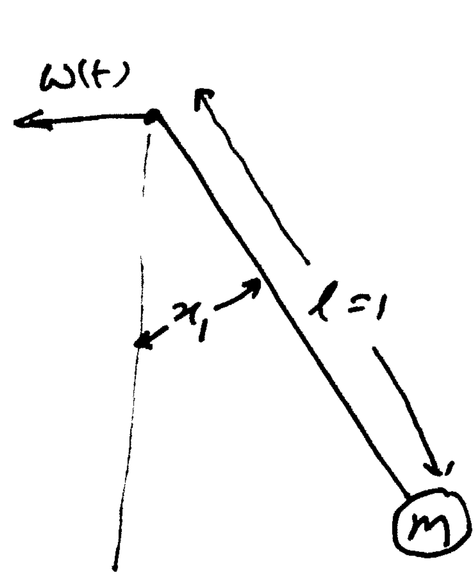

A pendulum is shown in the above figure where the rod has unit length, m is the mass of the bob, $x_1(t)$ is the angle of the pendulum from the vertical. We make the usual assumption that the rod is rigid with zero mass and the pivot is frictionless.

Force $= -mg \sin x_1 = m\ddot{x}_1$

$$\therefore \boxed{\ddot{x}_1 = -g \sin x_1}$$

Next we assume that the pivot point is subject to a horizontal motion $w(t)$. This induces an acc^n $\ddot{w}(t)$. The modified eqn of motion is given by

$$\ddot{x}_1 = -g \sin x_1 - \ddot{\omega} \cos x_1 .$$

Taking x_1 & $\underset{\overset{\shortparallel}{x_2}}{\dot{x}_1}$ as the state variable we have

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -g \sin x_1 - \ddot{\omega} \cos x_1 .$$

Linearizing about $x_1 = x_2 = 0$ we get.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -g x_1 - \ddot{\omega} \end{aligned} = \begin{pmatrix} 0 & 1 \\ -g & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \ddot{\omega}$$

Assume $g = 1$, $\ddot{\omega} = -\cos t$ we obtain.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t .$$

Discuss solution to this problem

for $x_1(0) = x_1^{(0)}$ $x_2(0) = x_2^{(0)}$.