

Math 3350

Home Work 8

H.W.

①

①  $A = \begin{pmatrix} 1 & 3 & 7 \\ 2 & 5 & 8 \\ 4 & 0 & -7 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 2 & 5 \\ 4 & 4 & 9 \\ 7 & 6 & -11 \end{pmatrix}$

- (a) Calculate  $AB$  and  $BA$  and check if they are equal.
- (b) Calculate  $\det A$ ,  $\det B$  to remind yourself how determinants are calculated.
- (c) Calculate  $\det(\lambda I - A)$  and  $\det(\lambda I - B)$   
They are called the characteristic polynomials.
- (d) If you have access to a computer, calculate the roots of the characteristic polynomials.  
They are called eigenvalues.
- (e) Using a computer write down all eigenvalues and eigenvectors of  $A$ .

② (a) Show that the o.d.e.

$$\ddot{y} + 3\dot{y} + 2y = f(t)$$

can be written as.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} f(t)$$

where

$$x_1 = y(t) \text{ and } x_2 = \dot{y}(t)$$

(b) Calculate the characteristic polynomial and eigenvalues of  $A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$ .

(c) Also calculate the corresponding eigenvectors.

③ Define

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!} + \dots$$

(a) using a simple calculator calculate  $e^A$  by adding 7 or 8 terms. Does it seem to converge.

(b) use matlab (if you can) to calculate  $e^A$ . (3)

(4) Define a new set of variables.

$$z_1 = 2x_1 + x_2.$$

$$z_2 = -x_1 - x_2.$$

Show that

$$\dot{z}_1 = -z_1 + f(t) \quad (*)$$

$$\dot{z}_2 = -2z_2 - f(t)$$

(5) (a) Assume  $y(0) = 1$ ,  $\dot{y}(0) = 2$  solve for  $x_1(0)$ ,  $x_2(0)$  and hence  $z_1(0)$ ,  $z_2(0)$ .

(b) With the obtained initial conditions, solve (\*) for  $f(t) = e^{-5t}$  for  $z_1(t)$  and  $z_2(t)$ .

(c) Write down  $x_1(t)$  and  $x_2(t)$  from above.