Solutions. H.W. 2

Chapter 2-4 page 65

(2) $(2x+y)\,dx - (x+6y)\,dy = 0$

$M = 2x+y$, $\dfrac{\partial M}{\partial y} = 1$. Not exact.

$N = -(x+6y)$, $\dfrac{\partial N}{\partial x} = -1$

(6) $\left(\dfrac{y}{x^2} - 4x^3 + 3y\sin 3x\right)dx + \left(2y - \dfrac{1}{x} + \cos 3x\right)dy = 0$

$M = \dfrac{y}{x^2} - 4x^3 + 3y\sin 3x$

$M_y = \dfrac{1}{x^2} + 3\sin 3x$ Not exact

$N = 2y - \dfrac{1}{x} + \cos 3x$

$N_x = +\dfrac{1}{x^2} - 3\sin 3x$

(20)

$$\underbrace{\left(\frac{1}{x}+\frac{1}{x^2}-\frac{y}{x^2+y^2}\right)}_{M}dx+\underbrace{\left(ye^y+\frac{1}{x^2+y^2}\right)}_{N}dy=0$$

$$M_y=-\left[\frac{(x^2+y^2)-y2y}{(x^2+y^2)^2}\right]$$

$$=-\frac{x^2-y^2}{(x^2+y^2)^2}=\frac{y^2-x^2}{(x^2+y^2)^2}$$

$$N_x=-\frac{1}{(x^2+y^2)^2}2x=-\frac{2x}{(x^2+y^2)^2}$$

Not exact

(28) $M=6xy^3+\cos y \Rightarrow M_y=6x3y^2-\sin y$

$M_y=18xy^2-\sin y.$

$N=2kx^2y^2-x\sin y. \Rightarrow N_x=2ky^22x-\sin y$

$=4kxy^2-\sin y$

$\therefore k=\frac{18}{4}=\frac{9}{2}$ for exactness.

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$$M = x^2 + 2xy - y^2$$

$$N = y^2 + 2xy - x^2$$

$$M_y = 2x - 2y \qquad \text{Not exact.}$$

$$N_x = 2y - 2x$$

$$M = \frac{x^2 + 2xy - y^2}{(x+y)^2}$$

$$N = \frac{y^2 + 2xy - x^2}{(x+y)^2}$$

$$M_y = \frac{(x+y)^2\left[2x - 2y\right] - \left[x^2 + 2xy - y^2\right]2(x+y}{(x+y)^4}$$

$$N_x = \frac{(x+y)^2\left[2y - 2x\right] - \left[y^2 + 2xy - x^2\right]2(x+y}{(x+y)^4}.$$

we want

$$2(x+y)(x-y) - 2\cancel{(x+y)}(x^2+2xy-y^2).$$

$$= 2(x+y)(y-x) - 2\cancel{(x+y)}(y^2+2xy-x^2).$$

$$2(x^2-y^2) - 2(x^2+2xy-y^2).$$

$$= -2(x^2-y^2) - 2(y^2+2xy-x^2).$$

$$4(x^2-y^2) + 2\left[y^2+\cancel{2xy}-x^2-x^2-\cancel{2xy}+y^2\right].$$

$$= 0.$$

Hence we have exactness.

Need to solve.

$$\frac{x^2+2xy-y^2}{(x+y)^2}\,dx+\frac{y^2+2xy-x^2}{(x+y)^2}\,dy=0.$$

$$\frac{x^2+2xy-y^2}{(x+y)^2} = \frac{\partial f}{\partial x} \qquad \frac{y^2+2xy-x^2}{(x+y)^2} = \frac{\partial f}{\partial y}.$$

$$\frac{\partial f}{\partial x}=\frac{(x+y)^2-2y^2}{(x+y)^2}=1-2\frac{y^2}{(x+y)^2}.$$

$$f(x,y)=x+2y^2\,\frac{1}{x+y}+g(y).$$

$$\frac{\partial f}{\partial y}=4y\,\frac{1}{x+y}+2y^2\,\frac{-1}{(x+y)^2}+g'(y)$$

$$=\frac{(y+x)^2-2x^2}{(y+x)^2}$$

$$=1-2\frac{x^2}{(x+y)^2}$$

$$g'(y) = 1 - \frac{2x^2}{(x+y)^2} + \frac{2y^2}{(x+y)^2} - \frac{4y}{x+y}.$$

$$= \frac{x^2+2xy+y^2-2x^2+2y^2}{(x+y)^2} - \frac{4y}{x+y}$$

$$= \frac{3y^2-x^2+2xy-4y(x+y)}{(x+y)^2}.$$

$$= \frac{3y^2-x^2+2xy-4xy-4y^2}{(x+y)^2}$$

$$= \frac{-y^2-x^2-2xy}{(x+y)^2}.$$

$$= \frac{-(x+y)^2}{(x+y)^2} = -1.$$

$$g(y) = -y.$$

$$f(x,y) = x + \frac{2y^2}{x+y} - y$$

$$= (x-y) + \frac{2y^2}{x+y}.$$

$$= \frac{x^2+y^2}{x+y}.$$

Sol^n

$$f(x, y) = \text{Constant}$$

$$\therefore \boxed{x^2 + y^2 = c(x+y).}$$

(32) $$\underbrace{y(x+y+1)}_{M}\,dx + \underbrace{(x+2y)}_{N}\,dy = 0.$$

$$M_y = x + 2y + 1$$

$$N_x = 1$$

$$\frac{M_y - N_x}{N} = \frac{x+2y}{x+2y} = 1.$$

$$\frac{d\mu}{dx} = \mu. \quad \boxed{\mu(x) = e^x}$$

$$\boxed{y(x+y+1)e^x\,dx + (x+2y)e^x\,dy = 0}$$

$$\frac{\partial f}{\partial x} = y(x+y+1)e^x$$

$$\frac{\partial f}{\partial y} = (x+2y)e^x$$

$$f = x \cdot e^x y + \cancel{2}e^x \frac{y^2}{\cancel{2}} + g(x).$$

$$f(x,y) = e^x y[x+y] + g(x).$$

$$\frac{\partial f}{\partial x} = e^x y[1] + e^x y[x+y] + g'(x).$$

$$= e^x y(x+y+1) + g'(x) = y(x+y+1)e^x$$

$$g'(x) = 0.$$

$$g(x) = C.$$

$$f(x,y) = e^x y(x+y) + C$$

해는 $\boxed{e^x y(x+y) = C}$.

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$$M = x^2 + y^2 - 5 \qquad y(0) = 1.$$

$$N = -y - xy.$$

$$M_y = 2y. \qquad \cancel{N_x - M_y}$$

$$N_x = -y.$$

$$\frac{M_y - N_x}{N} = \frac{2y + y}{-y(1+x)} = \frac{3\cancel{y}}{-\cancel{y}(1+x)}$$

$$= -\frac{3}{1+x}.$$

$$\frac{d\mu}{dx} = -\frac{3}{1+x}\mu.$$

$$\int \frac{d\mu}{\mu} = -\int \frac{3}{1+x}\,dx. \Rightarrow \ln\mu = -3\ln(1+x) + C$$

$$\mu = e^{\ln \frac{1}{(1+x)^3}} e^c$$

$$\boxed{\mu = k \frac{1}{(1+x)^3}}$$

Multiplying by $\frac{1}{(1+x)^3}$ we have.

$$\frac{x^2+y^2-5}{(1+x)^3} dx - \frac{y\cancel{(1+x)}}{(1+x)^{\cancel{3}2}} dy = 0 .$$

$$\boxed{\frac{x^2+y^2-5}{(1+x)^3} dx - \frac{y}{(1+x)^2} dy = 0}$$

$$\frac{\partial f}{\partial y} = -\frac{y}{(1+x)^2}$$

$$f(x,y) = -\frac{1}{(1+x)^2} \frac{y^2}{2} + g(x) .$$

$$\frac{\partial f}{\partial x} = -\frac{y^2}{\cancel{2}}\left(-\cancel{2}\right)\frac{1}{(1+x)^3} + g'(x)$$

$$= \frac{y^2}{(1+x)^3} + g'(x) = \frac{x^2+y^2-5}{(1+x)^3}$$

$$g'(x) = \frac{x^2-5}{(1+x)^3}$$

$$x^2-5 = (x+1)^2 - 2x - 6$$

$$= (x+1)^2 - 2(x+1) - 8$$

$$\therefore \frac{x^2-5}{(x+1)^3} = \frac{(x+1)^2}{(x+1)^3} - 2\frac{(x+1)}{(x+1)^3} - 8\frac{1}{(x+1)^3}$$

$$= \frac{1}{x+1} - 2\frac{1}{(x+1)^2} - 8\frac{1}{(x+1)^3}$$

$$g(x) = \ln(x+1) + 2\frac{1}{(x+1)} + 4\frac{1}{(x+1)^2} + C$$

$f(x,y) =$

$$-\frac{y^2}{2(1+x)^2} + \ln(x+1) + \frac{2}{x+1} + \frac{4}{(x+1)^2} = c$$

$f(0,1) =$

$$-\frac{1}{2(1)^2} + \cancel{\ln 1}^{\,0} + \frac{2}{1} + \frac{4}{1} = c$$

$$c = 6 - \frac{1}{2} = 5\frac{1}{2} = \frac{11}{2}.$$

$\therefore$ Sol^{n}

$$-\frac{1}{2}\frac{y^2}{(1+x)^2} + \ln(1+x) + \frac{2}{x+1} + \frac{4}{(x+1)^2} = \frac{11}{2}$$