Mid Semester Exam #2 - Make Up

Math 3350: Higher Mathematics for Engineers and Scientists I

Fall 09 - Section 012

- Time allowed: 1 hour 20 minutes.
- This is an open book exam.
- Answer all questions.
- Show all the necessary work to earn full credit.
- Answers written on the test paper will not be graded.
- Please print your name on the first page of your answer scripts.
- Write your name on all the pages

---

(1) Solve for $y(t)$, where $\ddot{y}(t) + 2\dot{y}(t) + 2y(t) = e^{-t}$; $y(0) = \dot{y}(0) = 0$.

---

(2) Solve the following Bernoulli's equation:

$$\frac{dy}{dx} + x^n y = x^n y^2. \quad \text{Where } n \text{ is a fixed integer with } n \geq 1.$$

---

(3) (a) Verify if the form given below is exact:

$$e^x(\sin y + 2\cos y)\,dx + e^x(\cos y - 2\sin y)\,dy.$$

(b) Calculate $f(x, y)$ such that $df(x, y)$ is the above form.

Midterm I (Make up).

Answers.

① $\ddot{y} + 2\dot{y} + 2y = e^{-t}$.

- $y(0) = \dot{y}(0) = 0$

- char poly is $\lambda^2 + 2\lambda + 2$.

$= (\lambda + 1)^2 + 1$

Roots are at $-1 \pm i$

$y_h(t) = Ae^{-t}\sin t + Be^{-t}\cos t$.

- $y_p(t) = ce^{-t}$ ← from table in the book.

$\dot{y}_p(t) = -ce^{-t}$

$\ddot{y}_p(t) = ce^{-t}$

$\ddot{y}_p + 2\dot{y}_p + 2y_p = (c - 2c + 2c)e^{-t} = e^{-t}$

$c = 1$.

2

$$y_p(t) = e^{-t}$$

$$\therefore y(t) = e^{-t}(A\sin t + B\cos t) + e^{-t}.$$

$$y(0) = B + 1 = 0 \Rightarrow B = -1$$

$$\therefore y(t) = e^{-t}(A\sin t - \cos t) + e^{-t}.$$

$$= e^{-t}(A\sin t - \cos t + 1).$$

$$\dot{y}(t) = e^{-t}[A\cos t + \sin t]$$

$$- e^{-t}[A\sin t - \cos t + 1].$$

$$\dot{y}(0) = A = 0 \Rightarrow A = 0.$$

$$\therefore \boxed{y(t) = e^{-t}(1 - \cos t)}$$

(3) (a)

$$\frac{\partial}{\partial y}\left[e^x(\sin y + 2\cos y)\right]$$

$$= e^x\left[\cos y - 2\sin y\right]$$

$$\frac{\partial}{\partial x}\left[e^x(\cos y - 2\sin y)\right]$$

$$= e^x(\cos y - 2\sin y)$$

They are equal.

Hence the form is exact.

(b) $$\frac{\partial f}{\partial x} = e^x(\sin y + 2\cos y)$$

$$f(x,y) = e^x(\sin y + 2\cos y) + \phi(y)$$

$$\frac{\partial f}{\partial y} = e^x(\cos y - 2\sin y) + \phi'(y) = e^x(\cos y - 2\sin y).$$

$$\phi'(y) = 0 \Rightarrow \phi(y) = c.$$

$$\therefore f(x,y) = e^x(\sin y + 2\cos y) + \text{Const.}$$

(2) Ans:

Substitute

$$u = \frac{1}{y}$$

so that

$$\frac{du}{dy} = -\frac{1}{y^2}$$

$$\therefore \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx}$$

$$= -\frac{1}{y^2}\left[x^n y^2 - x^n y\right]$$

$$= -x^n + \frac{x^n}{y}$$

$$= -x^n + x^n u$$

$$= x^n[u-1]$$

$$\therefore \boxed{\frac{du}{u-1} = x^n dx}$$

$$\ln|(u-1)| = \frac{x^{n+1}}{n+1}.$$

$$\therefore u-1 = e^{\frac{x^{n+1}}{n+1}}$$

$$\therefore u = 1 + e^{\frac{x^{n+1}}{n+1}}$$

$$\therefore y(x) = \frac{1}{u(x)} = \frac{1}{1+e^{\frac{x^{n+1}}{n+1}}}$$