

Midterm I

① Consider the following equation

$$\frac{d^2 y}{dt^2} + 4y = \sin t$$

with $y(0) = 0$, $\dot{y}(0) = \frac{7}{3}$. Calculate $y(t)$.

② Solve the following Bernoulli's equation

$$\frac{dy}{dt} + y + \frac{1}{2} e^{-t} y^3 = 0$$

where $y(0) = 1$. Calculate $y(t)$.

③

(a) Show that the form

$$3 \cos 3x dx + \sin 3x dy$$

is not exact.

(b) Find a function $\mu(y)$ such that

$$\mu(y) 3 \cos 3x dx + \mu(y) \sin 3x dy$$

is exact.

(c) Calculate $f(x, y)$ such that $df(x, y)$ is the form in (b).