Math 3350.

Midterm II

and its solutions.

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(please print)

(1) We want to find two homogeneous solutions $y_1(x)$, $y_2(x)$ for the equation

$$\frac{d^2y}{dx^2} + 16\frac{dy}{dx} + 63y = 0$$

write

$$y(x) = A\,y_1(x) + B\,y_2(x)$$

and assume initial conditions given by

$$y(0) = 1\,, \quad y'(0) = 3$$

calculate A and B. Hence write down $y(x)$.

1 Ans:

Char. polynomial

$$\lambda^2 + 16\lambda + 63 = 0$$

$$\Rightarrow (\lambda + 7)(\lambda + 9) = 0$$

$$\Rightarrow \lambda = -7 \text{ or } -9$$

Hence

$$y_1(x) = e^{-7x}$$

$$y_2(x) = e^{-9x}$$

$$y(x) = Ae^{-7x} + Be^{-9x}$$

$$y(0) = A + B = 1$$

$$y'(0) = -7Ae^{-7x} - 9Be^{-9x}\Big|_{x=0}$$

$$= -7A - 9B = 3$$

$$\boxed{\begin{aligned} A + B &= 1 \\ 7A + 9B &= -3 \end{aligned}}$$

$$A = 6$$

$$B = -5$$

$$\boxed{y(x) = 6e^{-7x} - 5e^{-9x}}$$

(2) We want to find two homogeneous solutions $y_1(x), y_2(x)$ for the equation

$$\frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + 25y = 0$$

Write

$$y(x) = A\,y_1(x) + B\,y_2(x)$$

and assume initial conditions given by

$$y(0) = 0, \quad y'(0) = 12$$

Calculate A and B. Hence write down $y(x)$.

(2) Ans:

Char. polynomial

$$\lambda^2 - 6\lambda + 25 = 0$$

$$\Rightarrow (\lambda - 3)^2 + 4^2 = 0$$

$$\Rightarrow (\lambda - 3)^2 = (4i)^2$$

$$\Rightarrow \lambda = 3 \pm 4i$$

$$\therefore y_1(x) = e^{3x} \sin 4x$$

$$y_2(x) = e^{3x} \cos 4x$$

$$y(x) = e^{3x}\left[A \sin 4x + B \cos 4x\right].$$

$$y(0) = B = 0$$

$$\therefore \ y(x) = A e^{3x} \sin 4x$$

$$y'(x) = A e^{3x} 4 \cos 4x$$
$$+ 3 A e^{3x} \sin 4x$$

$$y'(0) = 4A = 12 \implies A = 3.$$

$$\therefore \ y(x) = 3 e^{3x} \sin 4x.$$

(3) We are interested in obtaining a particular solution $y_p(x)$ of

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = -\sin 5x.$$

The homogeneous solution $y_h(x)$ is given as

$$y_h(x) = Ae^{-x} + Be^{-2x}.$$

(a) calculate the particular solution $y_p(x)$.

(b) Write down

$$y(x) = Ae^{-x} + Be^{-2x} + y_p(x).$$

choose initial condition

$$y(0) = \frac{15}{15^2 + 23^2}$$

$$y'(0) = \frac{5.23}{15^2 + 23^2} - 1$$

Calculate A and B from the above data.

(c) Hence write down $y(x)$.

——— x ———

(3) Ans:

$$y_p(x) = C\sin 5x + D\cos 5x.$$

$$y_p'(x) = 5C\cos 5x - 5D\sin 5x.$$

$$y_p''(x) = -25C\sin 5x - 25D\cos 5x.$$

Since

$$y_p'' + 3y_p' + 2y_p = -\sin 5x$$

we have

$$[2C - 15D - 25C]\sin 5x = -\sin 5x$$

$$+ (2D + 15C - 25D)\cos 5x$$

$$\Rightarrow -15D - 23C = -1$$

$$-23D + 15C = 0.$$

$$C = \frac{23}{15^2 + 23^2}$$

$$D = \frac{15}{15^2 + 23^2}$$

$$\therefore \; y_p(x) = \frac{23 \sin 5x + 15 \cos 5x}{15^2 + 23^2}$$

$$y(x) = y_h(x) + y_p(x)$$

$$= Ae^{-x} + Be^{-2x} + \frac{23 \sin 5x + 15 \cos 5x}{15^2 + 23^2}$$

$$y(0) = A + B + \frac{15}{15^2+23^2} = \frac{15}{15^2+23^2}$$

$$\Rightarrow A + B = 0$$

$$\Rightarrow \boxed{A = -B}$$

$$y'(x) = -Ae^{-x} - 2Be^{-2x} + \frac{23\times 5\cos 5x - 15\times 5\sin 5x}{15^2+23^2}$$

$$y'(0) = -A - 2B + \frac{23\times 5}{15^2+23^2} = \frac{5\times 23}{15^2+23^2} - 1$$

$$\therefore \boxed{A + 2B = 1}$$

Solving for A & B we get $A = -1, B = 1$

$y(x) =$

$$-e^{-x} + e^{-2x} + \frac{23\sin 5x + 15\cos 5x}{15^2 + 23^2}$$