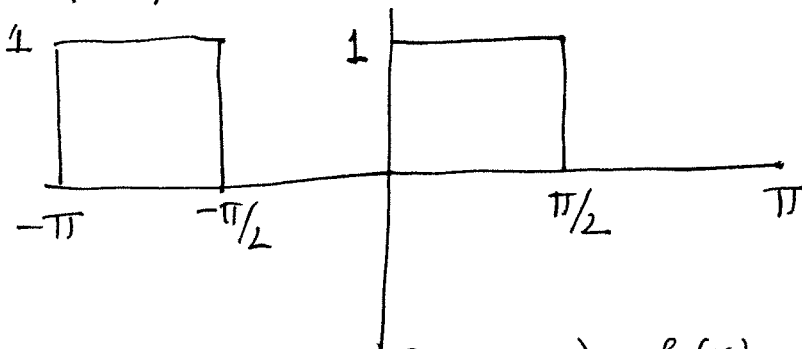# Home Work

(1) Let $f(x)$ be a function of period 2π and in the interval $-\pi < x < \pi$ are given by the formulas.

$$f(x) = \begin{cases} x^2 & \text{if } -\pi < x < 0 \\ -x^2 & \text{if } 0 < x < \pi. \end{cases}$$

Calculate the Fourier series expansion of x.

(2) Let $f(x)$ be the following f^n in the interval $-\pi$ to π

Assume that $f(x+\pi) = f(x)$ for all x.

(a) what is the period of f ??

(b) calculate the Fourier series expansion of f.

(3) We want to show that

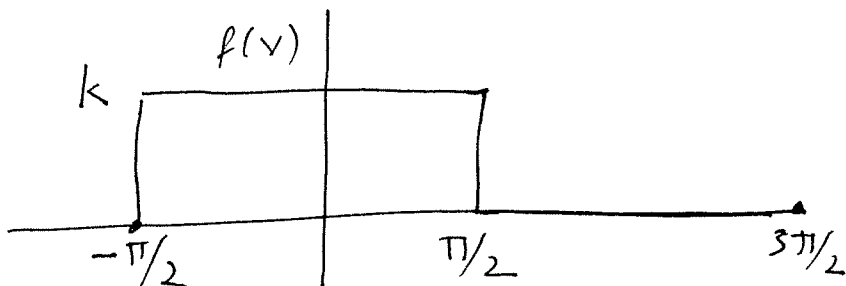$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}.$$

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots = \frac{\pi^2}{12}.$$

Calculate the Fourier series of

$$f(x) = \frac{x^2}{2} \qquad -\pi < x < \pi$$

Hence show the above results.

(4)

$f(v)$

k

$-\frac{\pi}{2}$ $\frac{\pi}{2}$ $\frac{3\pi}{2}$

Extending $f(x)$ to an even f^n, calculate the Fourier series expansion of this f^n. Hence show that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$