Math 3350

H.W. 4 (answers)

1 Ans:

char polynomial is

$$4\lambda^2 - 4\lambda - 3 = 0$$

$$\Rightarrow \lambda = \frac{4 \pm \sqrt{16+48}}{8}$$

$$= \frac{4 \pm \sqrt{64}}{8}$$

$$= \frac{4 \pm 8}{8} = \frac{\cancel{12}^{3}}{\cancel{8}_{2}}, -\frac{\cancel{4}^{1}}{\cancel{8}_{2}}$$

$$= 3/2, -1/2.$$

2

$$y_h(x) = ae^{3/2 x} + be^{-1/2 x}$$

$$y_h(-2) = e = ae^{3/2(-2)} + be^{-1/2(-2)}$$

$$= ae^{-3} + be^{1} = e$$

$$\boxed{ae^{-3} + be = e} \quad (1)$$

$$y_h'(x) = \frac{3}{2}ae^{3/2 x} - \frac{b}{2}e^{-\frac{1}{2}x}.$$

$$y_h'(-2) = -\frac{e}{2} = \frac{3}{2}ae^{-3} - \frac{b}{2}e$$

$$\boxed{3ae^{-3} - be = -e} \quad (2)$$

Solving (1) and (2) we have

$$4be = 4e \Rightarrow \boxed{b = 1}$$

$$\therefore ae^{-3} = 0 \Rightarrow a = 0$$

$$\therefore\ y_h(x) = e^{-x/2}$$

3

2 Ans:

char polynomial is.

$$4\lambda^2 - 4\lambda + 3 = 0$$

$$\Rightarrow \lambda = \frac{4 \pm \sqrt{16 - 48}}{8}$$

$$= \frac{4 \pm \sqrt{16 \times 2(-1)}}{8}$$

$$= \frac{4 \pm 4\sqrt{2}\, i}{8}$$

$$= \frac{1}{2} \pm \frac{\sqrt{2}}{2} i$$

$$= \frac{1}{2} \pm \frac{1}{\sqrt{2}} i$$

$$y_h(x) =$$

$$e^{\frac{1}{2}x}\left(A\sin\frac{1}{\sqrt{2}}x + B\cos\frac{1}{\sqrt{2}}x\right)$$

$$y_h(-2) = e = e^{-1}\left(A\sin\left(\frac{-2}{\sqrt{2}}\right) + B\cos\left(\frac{-2}{\sqrt{2}}\right)\right)$$

$$= \frac{1}{e}\left(-A\sin\sqrt{2} + B\cos\sqrt{2}\right)$$

$$B\cos\sqrt{2} - A\sin\sqrt{2} = e^2 \quad (3)$$

$$y_h'(x) = \frac{1}{2}e^{\frac{1}{2}x}\left(A\sin\frac{x}{\sqrt{2}} + B\cos\frac{x}{\sqrt{2}}\right) = y_h(x)$$

$$+ e^{x/2}\left(\frac{1}{\sqrt{2}}A\cos\frac{x}{\sqrt{2}} - \frac{1}{\sqrt{2}}B\sin\frac{x}{\sqrt{2}}\right)$$

$$y_h'(-2) = \frac{1}{2}e + \frac{1}{e}\left[\frac{A}{\sqrt{2}}\cos\sqrt{2} + \frac{B}{\sqrt{2}}\sin\sqrt{2}\right]$$

$$= -\frac{e}{2}$$

$$\frac{(B\sin\sqrt{2} + A\cos\sqrt{2})}{e\sqrt{2}} = -e$$

$$\Rightarrow B\sin\sqrt{2} + A\cos\sqrt{2} = -\sqrt{2}\,e^2 \quad (4)$$

Solving (3) & (4) we get

$$A = -e^2\left(\sqrt{2}\cos\sqrt{2} + \sin\sqrt{2}\right)$$

$$B = e^2\left(\cos\sqrt{2} - \sqrt{2}\sin\sqrt{2}\right)$$

$\blacksquare$

5 Ans:-

char polynomial is.

$$\lambda^2 + 10\lambda + 25 = 0$$

$$\Rightarrow \lambda = \frac{-10 \pm \sqrt{100 - 100}}{2}$$

$$= -5 \pm 0$$

$\therefore$ Repeated root at $-5, -5$

$$y_h(x) = ae^{-5x} + bxe^{-5x}$$

— x —

To find particular solution we note that Ae^{-5x} would not be a candidate because it is a part of the homogeneous solution.

We try Axe^{-5x}, but it is also a part of the homogeneous solution.

Try

$$y_p(x) = A x^2 e^{-5x}$$

$$y_p'(x) = A x^2 (-5) e^{-5x} + A 2x e^{-5x}$$

$$= -5A x^2 e^{-5x} + 2Axe^{-5x}$$

$$y_p''(x) = -5A\left[x^2(-5)e^{-5x} + 2xe^{-5x}\right]$$

$$+2A\left[x(-5)e^{-5x} + e^{-5x}\right]$$

$$= 25Ax^2e^{-5x} - 10Axe^{-5x}$$

$$-10Axe^{-5x} + 2Ae^{-5x}$$

$$= 25Ax^2e^{-5x} - 20Axe^{-5x} + 2Ae^{-5x}$$

$$y_p'' + 10y_p' + 25y_p = e^{-5x}$$

$$\Rightarrow \cancel{25Ax^2e^{-5x}} - \cancel{20Axe^{-5x}} + 2Ae^{-5x}$$

$$-\cancel{50Ax^2e^{-5x}} + \cancel{20Axe^{-5x}}$$

$$\cancel{25Ax^2e^{-5x}}$$

$$= e^{-5x}$$

$$\Rightarrow 2A = 1 \Rightarrow A = 1/2$$

$$\therefore y_p(x) = \frac{x^2}{2}e^{-5x}$$

—— x ——

$$y(x) = ae^{-5x} + bxe^{-5x} + \frac{x^2}{2}e^{-5x}$$

Homogeneous + particular