

Math 3350

①

H.W. 4 (answers)

① Ans:

char polynomial is

$$4\lambda^2 - 4\lambda - 3 = 0$$

$$\Rightarrow \lambda = \frac{4 \pm \sqrt{16 + 48}}{8}$$

$$= \frac{4 \pm \sqrt{64}}{8}$$

$$= \frac{4 \pm 8}{8} = \frac{12}{8}, -\frac{4}{8}$$

$$= \frac{3}{2}, -\frac{1}{2}$$

$$y_h(x) = a e^{3/2 x} + b e^{-1/2 x}$$

(2)

$$y_h(-2) = e = a e^{3/2(-2)} + b e^{-1/2(-2)}$$

$$= a e^{-3} + b e^1 = e$$

$$\boxed{a e^{-3} + b e = e} \quad (1)$$

$$y_h'(x) = \frac{3}{2} a e^{3/2 x} - \frac{b}{2} e^{-1/2 x}$$

$$y_h'(-2) = -\frac{e}{2} = \frac{3}{2} a e^{-3} - \frac{b}{2} e$$

$$\boxed{3 a e^{-3} - b e = -e} \quad (2)$$

Solving (1) and (2) we have

$$4 b e = 4 e \Rightarrow \boxed{b = 1}$$

$$\therefore a e^{-3} = 0 \Rightarrow \boxed{a = 0}$$

$$\therefore y_h(x) = e^{-x/2}$$

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② Ans:

char polynomial is .

$$4\lambda^2 - 4\lambda + 3 = 0$$

$$\Rightarrow \lambda = \frac{4 \pm \sqrt{16 - 48}}{8}$$

$$= \frac{4 \pm \sqrt{16 \times 2(-1)}}{8}$$

$$= \frac{4 \pm 4\sqrt{2}i}{8}$$

$$= \frac{1}{2} \pm \frac{\sqrt{2}}{2}i$$

$$= \frac{1}{2} \pm \frac{1}{\sqrt{2}}i$$

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$$y_h(x) =$$

$$e^{\frac{1}{2}x} \left(A \sin \frac{1}{\sqrt{2}} x + B \cos \frac{1}{\sqrt{2}} x \right)$$

$$y_h(-2) = e = e^{-1} \left(A \sin \left(\frac{-2}{\sqrt{2}} \right) + B \cos \left(\frac{-2}{\sqrt{2}} \right) \right)$$

$$= \frac{1}{e} \left(A \sin \sqrt{2} + B \cos \sqrt{2} \right)$$

$$B \cos \sqrt{2} - A \sin \sqrt{2} = e^2 \quad (3)$$

$= y_h(x)$

$$y_h'(x) = \frac{1}{2} e^{\frac{1}{2}x} \left(A \sin \frac{x}{\sqrt{2}} + B \cos \frac{x}{\sqrt{2}} \right) + e^{\frac{x}{2}} \left(\frac{1}{\sqrt{2}} A \cos \frac{x}{\sqrt{2}} - \frac{1}{\sqrt{2}} B \sin \frac{x}{\sqrt{2}} \right)$$

$$y_h'(-2) = \frac{1}{2} e + \frac{1}{e} \left[\frac{A}{\sqrt{2}} \cos \sqrt{2} + \frac{B}{\sqrt{2}} \sin \sqrt{2} \right] = -\frac{e}{2}$$

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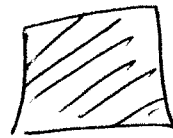
$$\frac{(B \sin \sqrt{2} + A \cos \sqrt{2})}{e \sqrt{2}} = -e$$

$$\Rightarrow (B \sin \sqrt{2} + A \cos \sqrt{2}) = -\sqrt{2} e^2 \quad (4)$$

Solving (3) & (4) we get

$$A = -e^2 (\sqrt{2} \cos \sqrt{2} + \sin \sqrt{2})$$

$$B = e^2 (\cos \sqrt{2} - \sqrt{2} \sin \sqrt{2})$$



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5) Ans: -

char polynomial is.

$$\lambda^2 + 10\lambda + 25 = 0$$

$$\Rightarrow \lambda = \frac{-10 \pm \sqrt{100 - 100}}{2}$$

$$= -5 \pm 0$$

\therefore Repeated root at $-5, -5$.

$$y_h(x) = a e^{-5x} + b x e^{-5x}$$

To find particular solution we note that $A e^{-5x}$ would not be a candidate because it is a part of the homogeneous solution.

(8)

We try $Ax e^{-5x}$, but it is also
a part of the homogeneous solution.

Try

$$y_p(x) = A x^2 e^{-5x}$$

$$y_p'(x) = A x^2 (-5) e^{-5x} + A 2x e^{-5x}$$
$$= -5A x^2 e^{-5x} + 2A x e^{-5x}$$

$$y_p''(x) = -5A [x^2 (-5) e^{-5x} + 2x e^{-5x}]$$
$$+ 2A [x (-5) e^{-5x} + e^{-5x}]$$

$$= 25A x^2 e^{-5x} - 10A x e^{-5x}$$
$$- 10A x e^{-5x} + 2A e^{-5x}$$

$$= 25A x^2 e^{-5x} - 20A x e^{-5x} + 2A e^{-5x}$$

(9)

$$y_p'' + 10y_p' + 25y_p = e^{-5x}$$

$$\begin{aligned} \Rightarrow & \cancel{25A} x^2 e^{-5x} - \cancel{20A} x e^{-5x} + 2A e^{-5x} \\ & - \cancel{50A} x^2 e^{-5x} + \cancel{20A} x e^{-5x} \\ & \cancel{25A} x^2 e^{-5x} + \cancel{25A} \\ & = e^{-5x} \end{aligned}$$

$$\Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2}$$

$$\therefore y_p(x) = \frac{x^2}{2} e^{-5x}$$

$$y(x) = a e^{-5x} + b x e^{-5x} + \frac{x^2}{2} e^{-5x}$$

Homogeneous + particular