Homework #5 - Math 2360: Linear Algebra

(1) Let V be the set of all ordered pairs of real numbers of the form (x_1, x_2), with the additon defined by

$$(x_1, x_2) \oplus (y_1 + y_2) = (x_1y_1 - x_2y_2, x_1y_2 + x_2y_1)$$

and scalar multiplication by

$$\alpha \odot (x_1, x_2) = (\alpha + x_1, x_2).$$

Is V a vector space with these operations? Justify your answer.

(2) Determine if W is a subspace of V for the pairs V and W given below.

V	W
$\mathbb{R}^4$	$\{(x_1, x_2, x_3, x_4) : x_1 + x_2 + x_3 + x_4 = 0\}$
$\mathbb{R}^3$	$\{(x_1, x_2, x_3) : x_1 + x_2 + x_3 = 1\}$
$\mathbb{R}^2$	$\{(x_1, x_2) : x_1 = 3x_2\}$
$\mathbb{R}^3$	$\{(x_1, x_2, x_3) : x_1 = x_2 = x_3\}$
Set of 2×2 matrices	Upper triangular matrices
Set of 2×2 matrices	Matrices of the form $\begin{pmatrix} 0 & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$
Polynomials $(a_2x^2 + a_1x^1 + a_0)$	Polynomials $(ax^2 + ax^1 + a)$

(3) (a) Determine if the elements given in the first column are linearly independant in the space given in the second column.
(b) Indicate if the elements would span the given space.
(c) In each case indicate the dimension of the span of the elements.
(d) Give *a natural* basis that spans the sub space spanned by the given elements in the first column.

Elements	Space
$\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \end{pmatrix}$	$\mathbb{R}^2$
$\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$	$\mathbb{R}^3$
$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	$\mathbb{R}^3$
$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	$\mathbb{R}^3$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	Space of 2×2 matrices