Math 2360: Linear Algebra
Fall 2009 - Section 004
Final Exam

- **Exam time: 7.30 am - 10.00 am (2 hours and 30 minutes)**
- This is a closed book exam.
- Answer all ten (10) questions.
- Show all the necessary work to earn full credit.
- Answers written on the test paper will not be graded.
- Please print your name on the first page of your answer book.
- Please write your answers clearly within the space provided in the answer book.

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(1) Solve the following pair of equations in 4 variables:

$$\begin{aligned} x_1 + 2x_3 - x_4 &= 2 \\ 2x_1 + x_2 + 2x_4 &= 1 \end{aligned}$$

Use either echelon form or reduced row echelon form.

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(2) Let

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 0 & 0 \end{pmatrix}$$

Calculate AB and BA and find out if they are equal.

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(3) Let A be the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 2 & 1 & 0 \end{pmatrix}$$

Calculate A^{-1} of this matrix.

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(4) Find a basis for the null space of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{pmatrix}$$

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(5) For the matrix in problem 4, check
  (a) if the rows are linearly independant
  (b) if yes, go to the next problem; if not, find which rows are linearly independant.

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(6) Consider the following vectors in $\mathbb{R}^5$

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 3 \\ 2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

(a) Are the vectors v_1, v_2 and v_3 linearly independant ?
(b) Find a vector which is not in span $\{v_1, v_2, v_3\}$.

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(7) Calculate the determinant of the following matrix A:

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 2 & 0 & 1 & 0 \\ 3 & 4 & 1 & 7 \\ -2 & 0 & 0 & -1 \end{pmatrix}$$

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(8) Let A be the 2×2 matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

(a) Write down $\det A$.
(b) Write down trace A.
(c) Write down $I + A$, where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
(d) Write down $\det(I + A)$.
(e) Verify if it is true that $\det(I + A) = 1 + \text{trace } A + \det A$.

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(9) Using the matrix A in problem 7, consider the linear equation

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 2 & 0 & 1 & 0 \\ 3 & 4 & 1 & 7 \\ -2 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Calculate x_2 using Cramer's rule.

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(10) Let L be a map from $\mathbb{R}^3$ to $\mathbb{R}^3$ described as follows:

$$L : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$

$$L \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_1 \\ x_2 \end{pmatrix}$$

(a) Is L a linear transformation ?
(b) Calculate $L(L(x))$.
(c) Find a matrix A such that $L(x) = Ax$.
