

Mid term II  
(make up).

① Calculate determinant of the matrix

$$\begin{pmatrix} 3 & 5 & 7 \\ 8 & 1 & 9 \\ 2 & 4 & 6 \end{pmatrix} \quad (10 \text{ pts})$$

② Verify if the following three vectors are linearly independent

$$\begin{pmatrix} 1 \\ 2 \\ 5 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 6 \\ 7 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 8 \\ 23 \end{pmatrix} \quad (10 \text{ pts})$$

③ Find a basis of the null space of the matrix

$$\begin{pmatrix} 3 & 4 & 1 \\ 2 & 1 & -1 \\ 3 & 4 & 1 \end{pmatrix} \quad (10 \text{ pts})$$

④ Find a basis of the column space of the matrix in ③. (10 pts)

⑤ Let  $V$  be an ordered pair of real numbers  $(x, y)$  with the following definition of addition and scalar multiplication

$$(x_1, y_1) \oplus (x_2, y_2) = (y_1 + y_2, x_1 + x_2)$$

$$\alpha \odot (x, y) = (\alpha y, \alpha x)$$

Consider each of the axioms of a vector space and verify which of the axioms are satisfied/not satisfied. (10 pts)