

Mid Semester Exam #2
Math 2360: Linear Algebra
Fall 09 - Section 004

- Time allowed: 1 hour 20 minutes.
 - This is a closed book exam.
 - Answer all questions.
 - Show all the necessary work to earn full credit.
 - Answers written on the test paper will not be graded.
 - Please print your name on the first page of your answer scripts.
 - Write your name on all the pages
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(1) Verify if the following vectors are linearly independent in \mathbb{R}^4 . State your reasons clearly.

$$\begin{pmatrix} 1 \\ 1 \\ 2 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 3 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 2 \\ -2 \\ 2 \end{pmatrix}.$$

(2) (a) A square matrix A is such that $A^3 = A$. What are the possible choices of $\det A$?

(b) Let R be a $n \times n$ matrix such that $RR^T = I$. Let A and B be two $n \times n$ matrices such that $B = RAR^T$. Show that $\det B = \det A$.

(3) (a) Let $V = \{(x_1, x_2, x_3, x_4) : x_1 + x_2 + x_3 = 0\} \subset \mathbb{R}^4$. Using addition and scalar multiplication in \mathbb{R}^4 , is V a vector space?

(b) Let $U = \{(x_1, x_2) : |x_1| = |x_2|\} \subset \mathbb{R}^2$. Using addition and scalar multiplication in \mathbb{R}^2 , is U a vector space?

(4) Let $N = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 4 & -1 & 3 & -5 \end{pmatrix}$. Describe the null space of N as a span of a set of vectors.
