

MATH 1352: CALCULUS II – Section 030

MID SEMESTER EXAM III

1 hour 20 minutes

- All calculations have to be from ground up. Show all work for full credit.
- The use of calculators, textbooks, class notes or mutual consultation is not allowed.
- Answers on the question paper will not be accepted.
- Clearly write your name on the answer sheet.
- All questions are of equal weightage but may not be of equal difficulty.

1. Using L'Hôpital's rule and integration by parts, evaluate the following integral, if it exists:

$$\int_0^{\infty} x e^{-3x} dx.$$

2. Calculate the volume of the Gabriel's Horn given by:

$$V = \pi \int_1^{\infty} \frac{1}{x^2} dx.$$

3. Calculate the following integral or show that it diverges:

$$\int_1^4 \frac{1}{x-2} dx.$$

4. Calculate $\frac{1}{4} + \left(\frac{1}{4}\right)^4 + \left(\frac{1}{4}\right)^7 + \left(\frac{1}{4}\right)^{10} + \dots$, if it exists or show that it does not exist.

5. Verify if the following series converges or diverges:

$$\sum_{n=2}^{\infty} \frac{1}{n(n+1)}.$$

6. Using the integral test, verify if the following series converges or diverges:

$$\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}.$$

7. Verify if the following series converges or diverges:

$$\sum_{k=1}^{\infty} \frac{3^k}{k^2}.$$