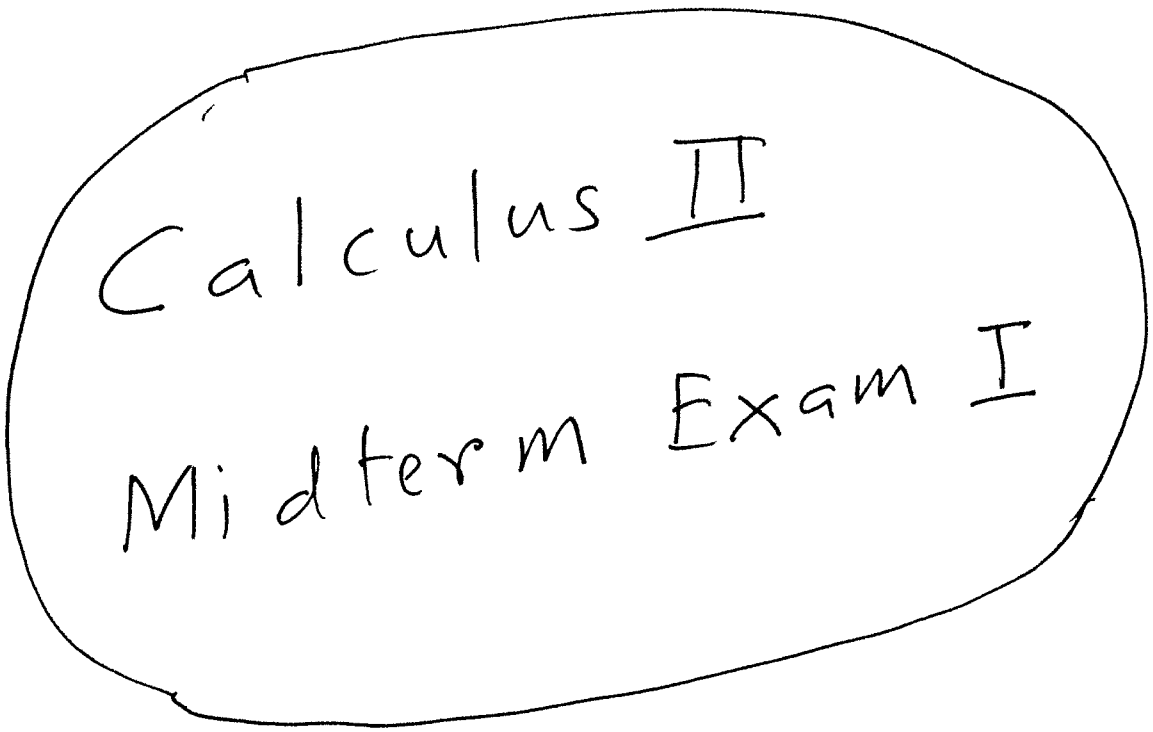
Calculus II
Midterm Exam I

(1) Ans:

Right curve is obtained from

$$(x+1)^2 + y^2 = 4$$

$$\Rightarrow x = \sqrt{4-y^2} - 1$$

Left curve is obtained from

$$(x-1)^2 + y^2 = 4$$

$$\Rightarrow x = -\sqrt{4-y^2} + 1$$

The circles intersect at $(0, \sqrt{3}), (0, -\sqrt{3})$

$$\text{Area} = 2\int_{-\sqrt{3}}^{\sqrt{3}} \left[\sqrt{4-y^2} - 1\right] dy \quad \text{(using symmetry)}$$

$$= 4\int_0^{\sqrt{3}} \sqrt{4-y^2} - 1 \, dy$$

$$= 4\left[2\sin^{-1}\frac{y}{2} + \frac{y}{2}\sqrt{4-y^2}\right]_0^{\sqrt{3}} - 4y\Big|_0^{\sqrt{3}}$$

$$= 8\sin^{-1}\frac{\sqrt{3}}{2} + 2\sqrt{3} - 4\sqrt{3}$$

$$= 8\sin^{-1}\frac{\sqrt{3}}{2} - 2\sqrt{3}$$

$$= 8\frac{\pi}{3} - 2\sqrt{3} = 4.91348$$

(2) Ans:

Cross sectional area =

$$\pi r^2$$

$$= \pi \left[1 - \frac{h^2}{4}\right]^2$$

$$= \pi \left[1 - \frac{h^2}{2} + \frac{h^4}{16}\right]$$

$$\Delta V = \pi \left[1 - \frac{h^2}{2} + \frac{h^4}{16}\right] \Delta h.$$

$$V = \int_0^2 \Delta V = \int_0^2 \pi \left[1 - \frac{h^2}{2} + \frac{h^4}{16}\right] dh.$$

$$= \pi \left[h - \frac{h^3}{6} + \frac{h^5}{80}\right]_0^2$$

$$= \pi \left[2 - \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{6}}} + \frac{\overset{2}{\cancel{32}}}{\underset{5}{\cancel{80}}}\right]$$

$$= \pi \left[2 - \frac{4}{3} + \frac{2}{5}\right] = \frac{30 - 20 + 6}{15} \pi$$

$$= \frac{16}{15} \pi$$

(3) Ans:

$$r^2 = a^2 \cos 2\theta$$

$$2r\frac{dr}{d\theta} = 2a^2(-\sin 2\theta)\,d\theta$$

$$\Rightarrow \frac{dr}{d\theta} = \frac{2a^2}{2r}(-\sin 2\theta)$$

$$= -\frac{a^2 \sin 2\theta}{a\sqrt{\cos 2\theta}} = -a\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$$

$$\left(\frac{dr}{d\theta}\right)^2 = a^2\frac{\sin^2 2\theta}{\cos 2\theta}$$

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = a^2\left[\cos 2\theta + \frac{\sin^2 2\theta}{\cos 2\theta}\right]$$

$$= a^2\frac{\cos^2 2\theta + \sin^2 2\theta}{\cos 2\theta}$$

$$= \frac{a^2}{\cos 2\theta}$$

$$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \frac{a}{\sqrt{\cos 2\theta}}$$

$$2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$$

$$= 2\pi a^2 \sin\theta$$

$$S = \int_0^{\pi/4} 2\pi a^2 \sin\theta \, d\theta$$

$$= -2\pi a^2 \cos\theta \Big|_0^{\pi/4}$$

$$= -2\pi a^2 \left[\cos\frac{\pi}{4} - \cos 0\right]$$

$$= 2\pi a^2 \left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= (2 - \sqrt{2})\,\pi a^2$$

$$= 1.8403\, a^2$$