

Review problems
from vectors.

① Find an unit vector in the same direction as.

$$(2 \quad 5 \quad -7)$$

② Calculate distance between two points.

and $(2 \quad 3 \quad 5)$

$$(-1 \quad -7 \quad 21)$$

③ Calculate angle between two vectors.

$$(2 \quad 3 \quad 5) \text{ and } (-1 \quad -7 \quad 21)$$

④ Write down the equation of a line perpendicular to the vector

$$(2 \quad 7)$$

and passing through the point $(1 \quad 1)$

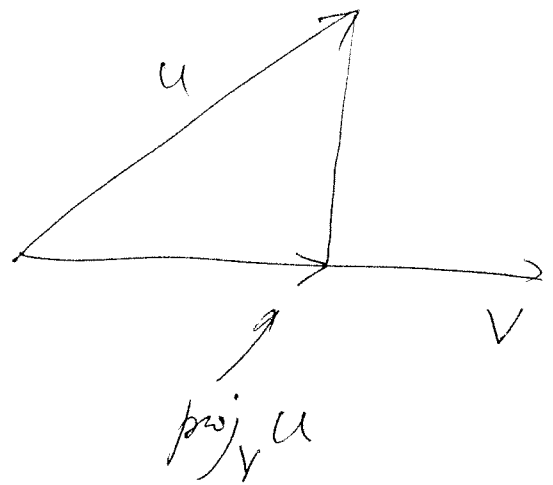
5) Find cross product of the vectors

$u = (2 \ 3 \ 5)$ ie calculate

$v = (-1 \ -7 \ 21)$ $u \times v$.

6) Calculate area of the triangle with corner points at $(0 \ 0 \ 0)$, $(2 \ 3 \ 5)$, $(-1 \ -7 \ 21)$.

7) Project the vector $(2 \ 3 \ 5)$ on the vector $(-1 \ -7 \ 21)$ ie find $proj_v u$.



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⑧ calculate

$$w = u - \text{proj}_v u$$

and show that

w is perpendicular to v

ie $w \cdot v = 0$.

⑨ Using $u \times v$ in problem 5 and angle θ in problem 3 show that

$$\|u \times v\| = \|u\| \|v\| \sin \theta.$$

⑩ write down the ~~sym~~ parametric form and symmetric form of a line in \mathbb{R}^3 that passes through the point $(1 \ 2 \ 3)$ and is parallel to the vector $(3 \ 7 \ -2)$.

Some other
Review problems

① consider the following geometric series

$$S = \frac{1}{7} + \frac{1}{7^3} + \frac{1}{7^5} + \frac{1}{7^7} + \frac{1}{7^9} + \dots$$

- Does the above series converge??
- Calculate S if it does?

② calculate the following limits?

- $\lim_{n \rightarrow \infty} \frac{n^2 + 3n + 4}{2n^2 + 5n - 9}$

- $\lim_{n \rightarrow \infty} \frac{n + 18}{n^2 + 2n + 6}$

③ calculate the following limits.

- $\lim_{t \rightarrow \infty} t^3 e^{-3t}$

if the limits exist.

- $\lim_{t \rightarrow \infty} t^3 e^{+3t}$

ie with proof.

④ Argue why the p -series

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$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \text{ diverges.}$$

and

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \text{ converges.}$$

⑤ Using ratio test show that the series

$$\sum_{k=1}^{\infty} \frac{k^k}{k!} \text{ converges.}$$

Recall that

$$\lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^k = e.$$

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6 calculate

$$\int_2^{\infty} t^2 e^{-3t} dt.$$

if it exists.

7 calculate

$$\int \sin^2 x \cos^2 x dx.$$

8 Using integration by parts, calculate

$$\int x \sin x dx.$$

④ ~~⑦~~
⑨ Using trigonometric substitution
calculate

$$\int \frac{x dx}{\sqrt{8-5x^2}}$$

and

$$\int \frac{dx}{\sqrt{8-5x^2}}$$

⑩ Find arc length of a curve
 $y = x^{3/2}$ on the interval $[0, 4]$.