First Midterm Answers

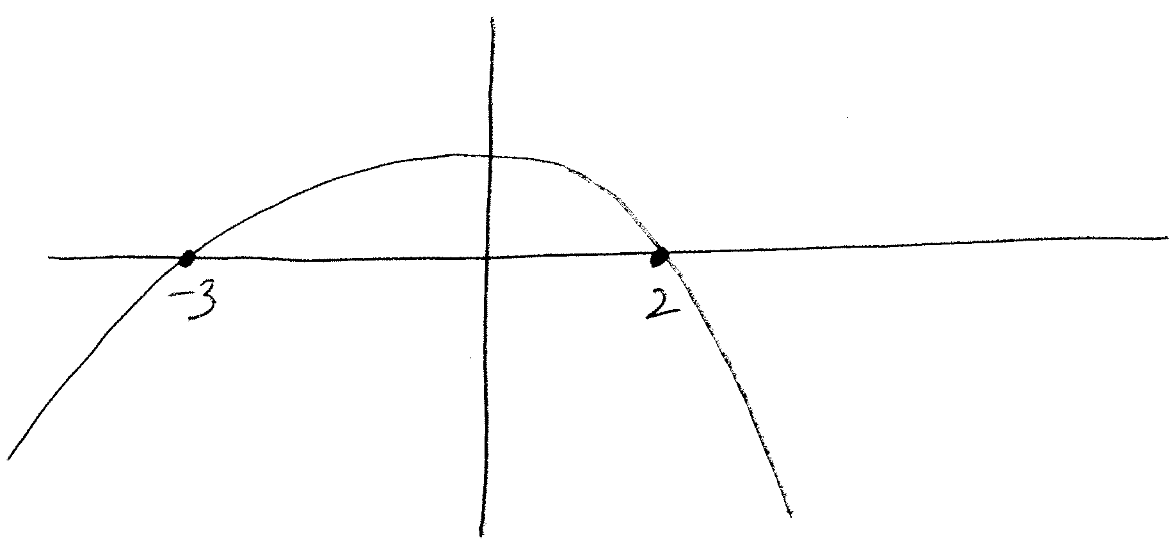
1 Ans:

Intersection of the parabola and the x-axis is given by

$$6 - x - x^2 = 0$$

$$\Rightarrow x^2 + x - 6 = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} = -3, 2$$



2

Area =

$$\int_{-3}^{2} (6 - x - x^2)\, dx$$

$$= 6x - \frac{x^2}{2} - \frac{x^3}{3} \Big|_{-3}^{2}$$

$$= \left(12 - 2 - \frac{8}{3}\right) - \left(-18 - \frac{9}{2} + \frac{\cancel{27}^{\,9}}{\cancel{3}}\right)$$

$$= 10 - \frac{8}{3} + 18 + \frac{9}{2} - 9$$

$$= 19 - \frac{8}{3} + \frac{9}{2}$$

$$= \frac{114 - 16 + 27}{6}$$

$$= \frac{141 - 16}{6} = \frac{125}{6}$$

2 Consider a horizontal cross section at a height h from the base. The side l of the square cross section is given as follows

$$\frac{l}{3-h} = \frac{2}{3}$$

$$\Rightarrow l = \left(1-\frac{h}{3}\right)2$$

$\therefore$ Area of the cross section $= \left(1-\frac{h}{3}\right)^2 4$

Volume of the elemental cross section of thickness

$$\Delta h = \left(1-\frac{h}{3}\right)^2 4\,\Delta h$$

Volume of the pyramid $=$

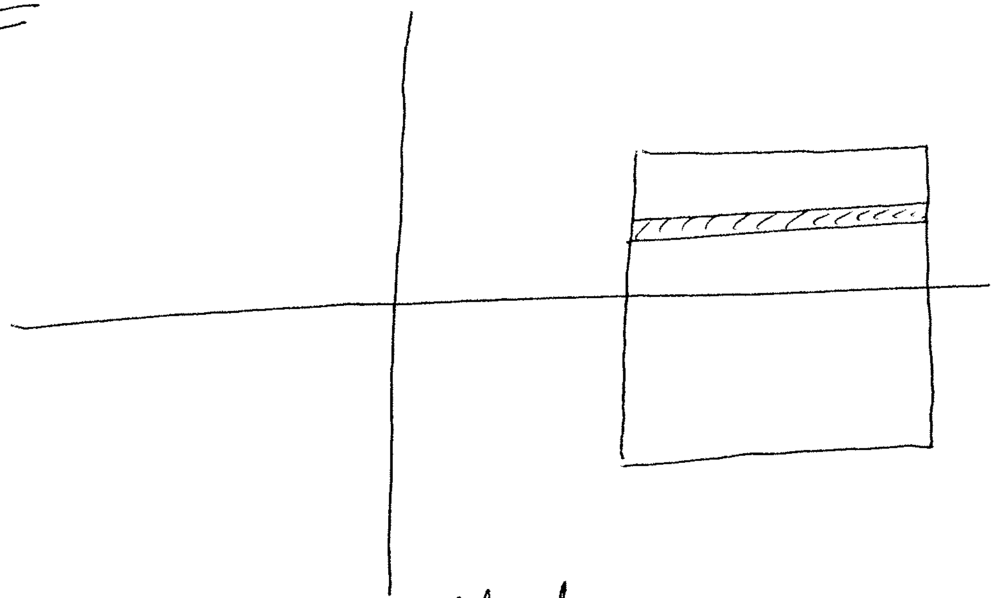
$$\int_0^3 \left(1-\frac{h}{3}\right)^2 4\,dh = 4\int_0^3 \left(1+\frac{h^2}{9}-\frac{2}{3}h\right)dh$$

$$= 4\left[h + \frac{h^3}{27} - \frac{\cancel{2}}{3}\frac{h^2}{\cancel{2}}\right]_0^3$$

$$= 4\left[3 + \frac{\cancel{27}}{\cancel{27}} - \frac{\overset{3}{\cancel{9}}}{\cancel{3}}\right]$$

$$= 4\left[1\right] = 4$$

3 Ans:



Using washer method

Area of the washer $= \pi(4^2 - 2^2)$

$= \pi(16 - 4) = 12\pi$.

Volume of the elemental washer of thickness $\Delta y =$

$12\pi \Delta y$.

Volume of the doughnut

$= \int_{-1}^{1} 12\pi \, dy = 12\pi y \Big|_{-1}^{1}$

$= 24\pi$.

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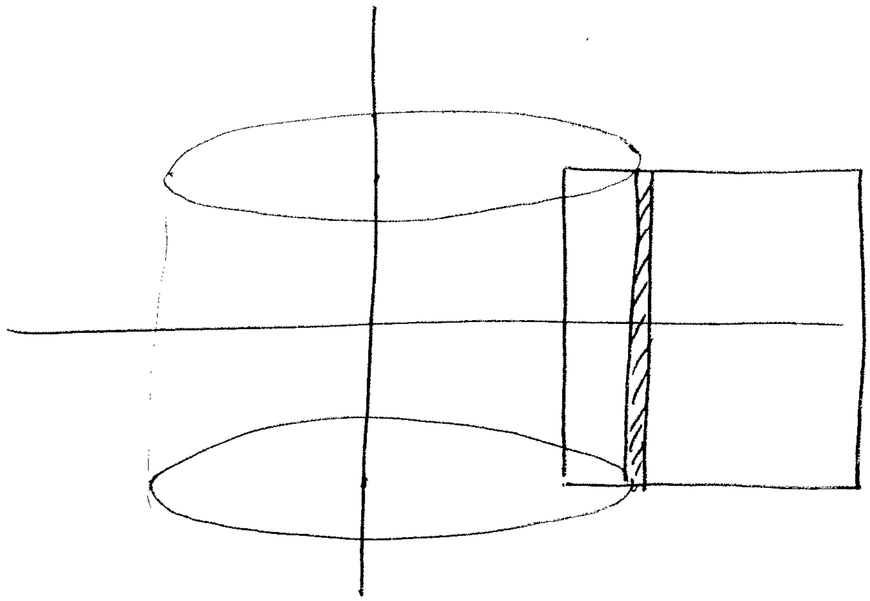


using shell method

surface area of the shell is

$$2\pi x \cdot 2 = 4\pi x$$

volume of the elemental shell is

$$4\pi x \, dx$$

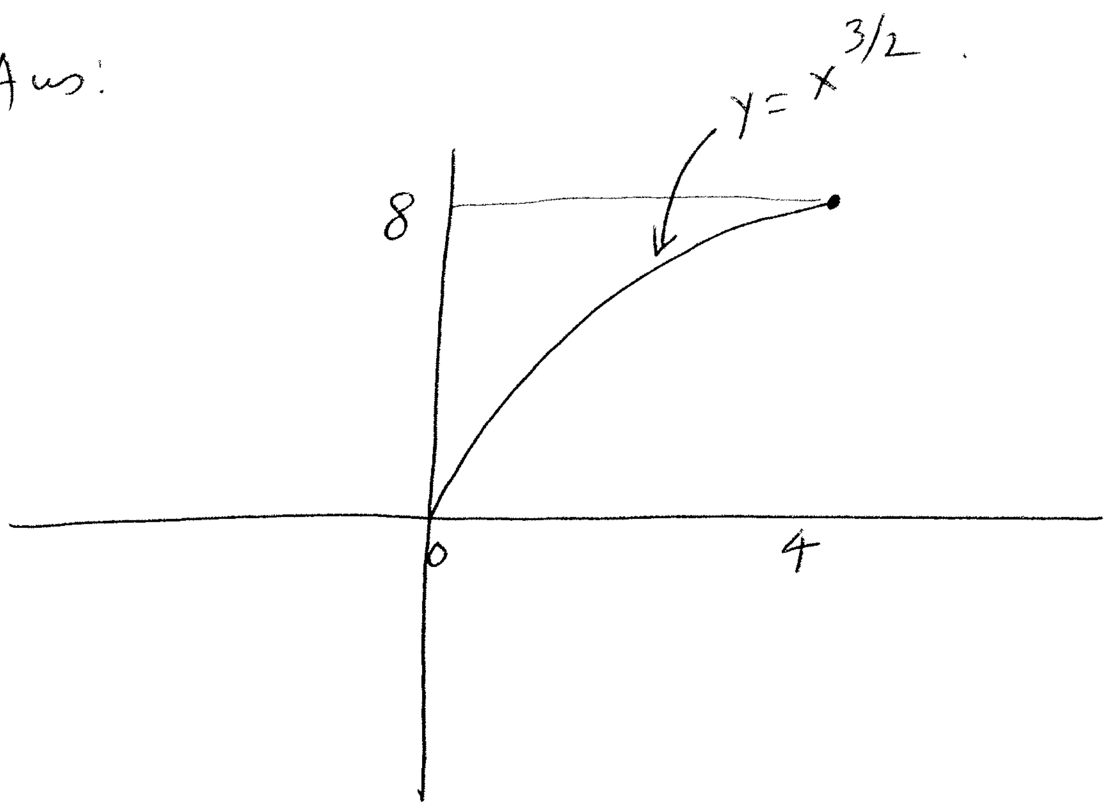
Volume of the doughnut

$$= \int_2^4 4\pi x \, dx = \overset{2}{\cancel{4}}\pi \frac{x^2}{\cancel{2}} \Bigg|_2^4$$

$$= \cancel{4\pi}\ 2\pi [16 - 4]$$

$$= 2\pi \, 12 = 24\pi$$

4) Ans:



$$L = \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{9}{4} x$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{9}{4} x}$$

$$\therefore L = \int_0^4 \sqrt{1 + 9/4\, x}\, dx = \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{3/2}\Bigg|_0^4$$

$$= \frac{8}{27}\left[10\sqrt{10} - 1\right]$$

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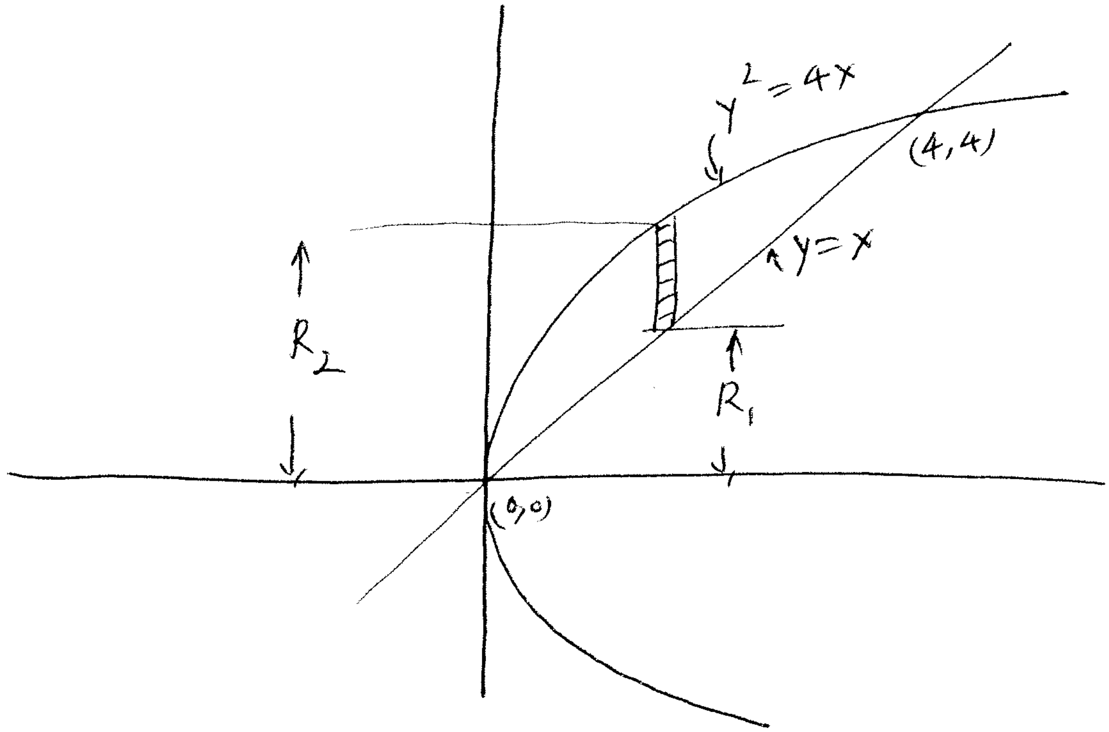

Area of the washer $= \pi\left(R_2^{\,2} - R_1^{\,2}\right)$

$= \pi\left(4x - x^2\right)$

Required volume =

$$\int_0^4 \pi\left(4x - x^2\right)dx = \pi\left[2x^2 - \frac{x^3}{3}\right]_0^4$$

$$= \pi\left[32 - \frac{64}{3}\right]$$

$$= \pi\left[\frac{96-64}{3}\right] = \frac{32}{3}\pi$$