Homework #9 - Vectors

1. Sketch the following vectors given in the component form and compute the magnitude.
[Remember that the length of a vector $x\,\boldsymbol{i} + y\,\boldsymbol{j}$ is given by $\sqrt{x^2 + y^2}$]

a. $1\,\boldsymbol{i} + 2\,\boldsymbol{j}$ b. $-4\,\boldsymbol{i} + 3\,\boldsymbol{j}$ c. $-5\,\boldsymbol{i} - 12\,\boldsymbol{j}$

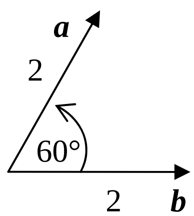
2. Find the vector (in component form) connecting each pair of points given.

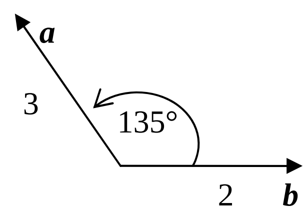
a. P = (-1,5) & Q = (9, 8)   b. P = (-1,0) & Q = (0, 0)   c. P = (10,5) & Q = (10, 8)

3. Find the values of x and y which satisfy the following equations.
[Hint: When do you say that two vectors equal?]

a. $(8 + x)\,\boldsymbol{i} + (8 + y)\,\boldsymbol{j} = (3x)\,\boldsymbol{i} + 0\,\boldsymbol{j}$ b. $(4 + x^2)\,\boldsymbol{i} + (8)\,\boldsymbol{j} = (4x)\,\boldsymbol{i} + (\log_e y)\,\boldsymbol{j}$

4. Find $\boldsymbol{a} + \boldsymbol{b}$ and $\boldsymbol{a} - \boldsymbol{b}$ in component form and demonstrate how the parallelogram law is used.





5. Use 'dot product' of vectors to find the angles between the given vectors:
[Remember that for two vectors $\boldsymbol{a} = a_1\,\boldsymbol{i} + a_2\,\boldsymbol{j}$ and $\boldsymbol{b} = b_1\,\boldsymbol{i} + b_2\,\boldsymbol{j}$, the dot product is given by $\boldsymbol{a.b} = |\boldsymbol{a}|.|\boldsymbol{b}|.\cos\theta = a_1a_2 + b_1b_2$, where θ is the angle from a to b]

i. $\boldsymbol{a} = 1\,\boldsymbol{i} + 2\,\boldsymbol{j}$ and $\boldsymbol{b} = 4\,\boldsymbol{i} + 3\,\boldsymbol{j}$ ii. $\boldsymbol{a} = 1\,\boldsymbol{i} + -4\,\boldsymbol{j}$ and $\boldsymbol{b} = -4\,\boldsymbol{i} + 3\,\boldsymbol{j}$

6. Using the dot product of vectors, find the angle between all the possible combinations of sides and diagonals of a cube of which each side is 1 unit in length.