(1) $\int \sqrt{7-4x^2}\,dx = \int 2\sqrt{\left(\frac{7}{4}-x^2\right)}\,dx$ $\quad x = \frac{\sqrt{7}}{2}\sin\theta$

$dx = \frac{\sqrt{7}}{2}\cos\theta\,d\theta$

$$= \int 2\sqrt{\left(\frac{7}{4}-\frac{7}{4}\sin^2\theta\right)}\,\frac{\sqrt{7}}{2}\cos\theta\,d\theta$$

$$= \sqrt{7}\left(\frac{\sqrt{7}}{2}\right)\int \cos^2\theta\,d\theta$$

using $\cos^2\theta = \frac{\cos 2\theta + 1}{2}$

$$= \frac{7}{2}\int \frac{\cos 2\theta + 1}{2}\,d\theta$$

$$= \frac{7}{4}\left[\frac{\sin 2\theta}{2} + \theta\right] + C$$

$$= \frac{7}{4}\left[\sin 2\left(\sin^{-1}\left(\frac{2}{\sqrt{7}}x\right)\right) + \sin^{-1}\left(\frac{2x}{\sqrt{7}}\right)\right] + C$$

(2) $\int \sqrt{7+4x^2}\,dx = \int \sqrt{4\left(\frac{7}{4}+x^2\right)}\,dx$

$$= 2\int \sqrt{\left(\frac{7}{4}+x^2\right)}\,dx$$

$x = \frac{\sqrt{7}}{2}\tan\theta$

$dx = \frac{\sqrt{7}}{2}\sec^2\theta\,d\theta$

$$= 2\int \sqrt{\frac{7}{4}+\frac{7}{4}\tan^2\theta}\;\frac{\sqrt{7}}{2}\sec^2\theta\,d\theta$$

$$= \frac{(\sqrt{7})\sqrt{7}}{2}\int \sec^3\theta\,d\theta$$

$$= \frac{7}{2}\int \sec^3\theta\,d\theta$$

$$= \frac{7}{2}\Big\{$$

$\int \sec^3\theta\,d\theta = \tan\theta\sec\theta - \int(\sec^2\theta - 1)\sec\theta\,d\theta$

$= \tan\theta\sec\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$

$\int \sec^3\theta\,d\theta = \frac{1}{2}\Big\{\tan\theta\sec\theta + \ln|\sec\theta + \tan\theta|\Big\}$ —

$$= \frac{7}{2} \left[\frac{1}{2}\left\{\tan\theta\sec\theta + \ln|\sec\theta + \tan\theta|\right\}\right\} + C$$

$$= \frac{7}{2} \left\{ \frac{1}{2}\left\{ \tan\left(\tan^{-1}\frac{2x}{\sqrt{7}}\right)\sec\left(\tan^{-1}\frac{2x}{\sqrt{7}}\right) + \ln\left|\sec\tan^{-1}\frac{2x}{\sqrt{7}} + \tan\tan^{-1}\left(\frac{2x}{\sqrt{7}}\right)\right|\right\} + C$$

③ $$-\int \sqrt{4x^2-7}\,dx = -2\int\sqrt{x^2-\frac{7}{4}}\,dx$$

Let $x = \sqrt{\frac{7}{2}}\sec\theta \qquad dx = \frac{\sqrt{7}}{2}\sec\theta\tan\theta\,d$

$$= -2\int\sqrt{\frac{7}{4}\sec^2\theta - \frac{7}{4}}\,\frac{\sqrt{7}}{2}\sec\theta\tan\theta\,d\theta$$

$$= -2\int\frac{7}{4}\sqrt{\sec^2\theta - 1}\,\sec\theta\tan\theta\,d\theta$$

$$= -2\,\frac{7}{4}\int\sec\theta\tan^2\theta\,d\theta$$

$$= -2\,\frac{7}{4}\int\sec\theta\,(\sec^2\theta - 1)\,d\theta$$

$$= -\frac{7}{2}\left\{\int\sec\theta\,d\theta - \int\sec^3\theta\,d\theta\right.$$

$$= -\frac{7}{2}\left\{\ln|\sec\theta+\tan\theta| - \frac{1}{2}\tan\theta\sec\theta - \frac{1}{2}\ln|\sec\theta+\tan\theta|\right\}$$

$$= -\frac{7}{2}\left\{\frac{1}{2}\ln|\sec\theta+\tan\theta| - \frac{1}{2}\sec\theta\tan\theta\right\}$$

$$= -\frac{7}{2}\left\{\frac{1}{2}\ln\left|\sec\sec^{-1}\frac{2x}{\sqrt{7}} + \tan\sec^{-1}\left(\frac{2x}{\sqrt{7}}\right)\right| - \frac{1}{2}\sec\sec^{-1}\frac{2x}{\sqrt{7}}\tan\sec^{-1}\frac{2}{\sqrt{\ }}\right.$$

(4) $\int \sqrt{3x^2+6x+8}\, dx$

$$3x^2+6x+8 = 3(x^2+2x)+8$$
$$= 3\left((x+1)^2\right) + 8 - 3$$
$$= 3(x+1)^2 + 5$$

ans.

$$= \int \sqrt{3(x+1)^2 + 5}\, dx$$
$$= \sqrt{3} \int \sqrt{(x+1)^2 + \frac{5}{3}}\, dx$$

Let $(x+1) = \sqrt{\frac{5}{3}} \tan\theta$

$$dx = \sqrt{\frac{5}{3}} \sec^2\theta\, d\theta$$

$$= \sqrt{3} \int \sqrt{\frac{5}{3}\tan^2\theta + \frac{5}{3}}\, \frac{\sqrt{5}}{\sqrt{3}} \sec^2\theta\, d\theta$$

$$= \sqrt{3} \int \sqrt{\frac{5}{3}} \sqrt{\tan^2\theta + 1}\, \frac{\sqrt{5}}{\sqrt{3}} \sec^2\theta\, d\theta$$

$$= \frac{\sqrt{5}\sqrt{5}}{\sqrt{3}} \int \sec^3\theta\, d\theta.$$

$$= \frac{5}{\sqrt{3}} \cdot \left[\frac{1}{2}\tan\theta\sec\theta + \frac{1}{2}\ln\left|\sec\theta + \tan\theta\right| \right] + C$$

$=$

⑤ $\int \sqrt{3x^2+6x-2}\ dx$

$$3x^2+6x-2 = 3(x^2+2x) - 2$$

$$= 3((x+1)^2) - 2 - 3$$

$$= 3(x+1)^2 - 5$$

$$\int \sqrt{3x^2+6x-2}\ dx = \int \sqrt{3(x+1)^2 - 5}\ dx$$

$$= \sqrt{3} \int \sqrt{(x+1)^2 - 5/3}\ dx$$

$$= \sqrt{3} \int \sqrt{(x+1)^2 - \frac{5}{3}}\ dx$$

$$(x+1) = \sqrt{\frac{5}{3}} \sec\theta$$

$$dx = \sqrt{\frac{5}{3}} \tan\theta \sec\theta\, d\theta$$

$$= \sqrt{3} \int \sqrt{\frac{5}{3}\sec^2\theta - \frac{5}{3}}\ \tan\theta \sec\theta\, d\theta$$

$$= \sqrt{5} \int \tan^2\theta \sec\theta\, d\theta.$$

$$= \sqrt{5} \int (\sec^2\theta - 1) \sec\theta\, d\theta$$

$$=$$

$$= \sqrt{5} \left\{ \int \sec^3\theta\, d\theta - \int \sec\theta\, d\theta \right\}$$

$$= \sqrt{5}\ \Big\{ \tfrac{1}{2} \tan\theta \sec\theta + \tfrac{1}{2} \ln|\sec\theta + \tan\theta|$$

$$- \ln|\sec\theta + \tan\theta| \Big\} + \text{Const}$$

$$= \sqrt{5}\ \Big\{ \tfrac{1}{2} \tan.\ \sec^{-1}\frac{\sqrt{3}(x+1)}{\sqrt{5}}\ \sec\ \sec^{-1}\frac{\sqrt{3}(x+1)}{\sqrt{5}}.$$

$$- \tfrac{1}{2} \ln\left| \sec\ \sec^{-1}\frac{\sqrt{3}}{\sqrt{5}}(x+1) + \tan\ \sec^{-1}\frac{\sqrt{3}}{\sqrt{5}}(x+1) \right| + \text{Cons}$$

⑥. $$\int \frac{1}{3x^2+6x+8}\, dx$$

$$3x^2+6x+8 = 3(x+1)^2+5.$$

~~$(x+1) =$~~

$$\int \frac{1}{3x^2+6x+8}\, dx = \int \frac{1}{3(x+1)^2+5}\, dx = \int \frac{1}{3\left[(x+1)^2+\frac{5}{3}\right]}$$

$$(x+1) = \frac{\sqrt{5}}{\sqrt{3}} \tan\theta \qquad dx = \frac{\sqrt{5}}{\sqrt{3}} \sec^2\theta\, d\theta$$

$$= \int \frac{1}{3\left(\frac{5}{3}\tan^2\theta + \frac{5}{3}\right)} \sec^2\theta\, d\theta$$

$$= \frac{1}{3}\int \frac{3}{5}\ \frac{1}{\sec^2\theta} \sec^2\theta\, d\theta = \frac{1}{5}\theta + \text{const}$$

$$= \frac{1}{5} \tan^{-1}\frac{\sqrt{3}(x+1)}{\sqrt{5}} + \text{const}$$

$$= \frac{5}{\sqrt{3}} \left[\frac{1}{2} \tan \tan^{-1} \frac{\sqrt{3}}{\sqrt{5}}(x+1) \sec \tan^{-1} \frac{\sqrt{3}}{\sqrt{5}}(x+1) \right.$$

$$\left. + \frac{1}{2} \ln \left| \sec \frac{\sqrt{3}}{\sqrt{5}}(x+1) + \tan \tan^{-1} \frac{\sqrt{3}}{\sqrt{5}}(x+1) \right| \right] + \text{Const}$$