HOME WORK 6

Solutions

① Expand

(a) $$\frac{3x+7}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+3)}$$

$$= \frac{A(x+3)+B(x+2)}{(x+2)(x+3)}$$

$$3x+7 = (A+B)\,x + 3A + 2B$$

Equating the coefficients We get

$$\left.\begin{aligned} A+B &= 3 \\ 3A+2B &= 7 \end{aligned}\right\} \Rightarrow \begin{aligned} A &= 1 \\ B &= 2 \end{aligned}$$

So $$\frac{3x+7}{(x+2)(x+3)} = \frac{1}{(x+2)} + \frac{2}{(x+3)}$$

(b) $$\frac{3x+7}{(x+2)^2(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x+3)}$$

$$= \frac{A(x+2)(x+3) + B(x+3) + C(x+2)^2}{(x+2)^2(x+3)}$$

$$= \frac{A(x^2+5x+6) + Bx+3B + C(x^2+4x+4)}{(x+2)^2(x+3)}$$

$$= \frac{(A+C)x^2 + (5A+B+4C)x + 6A+3B+4C}{(x+2)^2(x+3)}.$$

Equating the coefficients we get

$$\left.\begin{aligned} A+C &= 0 \\ 5A+B+4C &= 3 \\ 6A+3B+4C &= 7 \end{aligned}\right\} \Rightarrow \begin{aligned} A &= 2 \\ B &= 1 \\ C &= -2 \end{aligned}$$

So $$\frac{3x+7}{(x+2)^2(x+3)} = \frac{2}{(x+2)} + \frac{1}{(x+2)^2} - \frac{2}{(x+3)}$$

(e) $$\int \frac{3x+7}{(x+2)(x+3)}\,dx = \int \frac{1\,dx}{(x+2)} + \int \frac{2}{x+3}\,dx$$

$$= \ln|x+2| + 2\ln|x+3| + c$$; where c is an integratin Constant.

$$\int \frac{3x+7}{(x+2)^2(x+3)}\,dx = 2\int \frac{1}{(x+2)}\,dx + \int \frac{1\,dx}{(x+2)^2} - 2\int \frac{1}{(x+3)}\,dx$$

$$= 2\ln|x+2| - \frac{1}{(x+2)} - 2\ln|x+3| + c$$; c is an integratin Constant.

② $f(x) = e^{7x} \sin(9x)$

$g(x) = e^{7x} \cos(9x)$

$I_1 = \int f(x)\, dx$

$\int f(x)\, dx = \int e^{7x} \sin(9x)\, dx$

Let $u = e^{7x}$ $\quad dv = \sin(9x)\,dx \Rightarrow v = \frac{-\cos(9x)}{9}$

using integration by parts we get

$= e^{7x} \frac{(-\cos(9x))}{9} - \int - \frac{\cos(9x)\, 7 \cdot e^{7x}}{9}\, dx$

$= -\frac{e^{7x}\cos(9x)}{9} + \frac{7}{9}\underbrace{\int \cos(9x) e^{7x}\, dx}$

again using integration by parts we have.

$I_1 = \frac{-e^{7x}\cos(9x)}{9} + \frac{7}{9} I_2 \quad (1)$

$\int \cos(9x)\, e^{7x}\, dx = e^{7x}\frac{(+\sin 9x)}{9} - \underbrace{\int \frac{\sin 9x}{9}\, 7 \cdot e^{7x}\, dx}_{I_1}$

$I_2 = \frac{e^{7x}\sin(9x)}{9} - \frac{7}{9} I_1 \quad (2)$

(1), (2) gives

$I_1 = -\frac{e^{7x}\cos 9x}{9} + \frac{7}{9}\left(\frac{e^{7x}\sin(9x)}{9} - \frac{7}{9} I_1\right)$

$I_1\left(1 + \left(\frac{7}{9}\right)^2\right) = \frac{e^{7x}}{9}\left(\frac{7\sin 9x}{9} - \cos 9x\right)$

$I_1 = \frac{9e^{7x}}{130}\left(\frac{7\sin 9x}{9} - \cos(9x)\right) + C$; C is an integrating constant

$I_2 = \frac{9e^{7x}}{32}\left(\frac{7\cos 9x}{9} + \sin 9(x)\right) + C$; C is an integrating constant

$$I_1 = \frac{e^{7x}}{130}\left(7\sin 9x - 9\cos 9x\right) + C.$$

$$I_2 = \frac{e^{7x}}{32}\left(7\cos 9x - 9\sin 9x\right) + C$$