Math 1352

H. W. 5 (Answers)

Section 7.2

18 $\int_1^e x^3 \ln x \, dx$

$$\int_1^e \ln x \; x^3 \, dx =$$

$$\ln x \frac{x^4}{4}\Big|_1^e - \int_1^e \frac{1}{\cancel{x}} \frac{x^{\cancel{4}\,3}}{4} dx$$

$$= \frac{1}{4} x^4 \ln x \Big|_1^e - \int_1^e \frac{x^3}{4} dx$$

$$= \frac{1}{4} x^4 \ln x - \frac{1}{4}\frac{x^4}{4}\Big|_1^e$$

$$= \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 \Big|_1^e$$

$$= \frac{1}{4} e^4 \ln e - \frac{1}{16} e^4 - \frac{1}{4} \cancel{1 \ln 1}^{\,0} + \frac{1}{16}$$

$$= \frac{1}{4} e^4 - \frac{1}{16} e^4 + \frac{1}{16} = \frac{4-1}{16} e^4 + \frac{1}{16}$$

$$= \frac{3}{16} e^4 + \frac{1}{16}.$$

17 $\int_1^4 \sqrt{x} \ln x \, dx$

$$\int \ln x \, x^{1/2} \, dx$$

$$= \ln x \frac{x^{3/2}}{3/2} - \int \frac{1}{x} \frac{x^{3/2}}{3/2} \, dx$$

$$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{1/2} \, dx$$

$$= \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} \Big|_1^4$$

$$= \left[\frac{2}{3} 4\sqrt{4} \ln 4 - \frac{4}{9} 4\sqrt{4}\right] - \left[\frac{2}{3} 1 \cancel{\ln 1}^{0} - \frac{4}{9} 1\right]$$

$$= \frac{2}{3} 4.2 \ln 4 - \frac{4}{9} 4.2 + \frac{4}{9}.$$

$$= \frac{16}{3} \ln 4 - \frac{32}{9} + \frac{4}{9} = \frac{32}{3} \ln 2 - \frac{28}{9}$$

21 Ans:

$$\int_0^{\pi} e^{2x}\cos 2x\,dx$$

$$= \overset{0}{e^{2x}\frac{\sin 2x}{2}\Big|_0^{\pi}} - \int_0^{\pi} \cancel{2}e^{2x}\frac{\sin 2x}{\cancel{2}}\,dx$$

$$= -\int_0^{\pi} e^{2x}\sin 2x\,dx$$

$$= +e^{2x}\frac{\cos 2x}{2}\Big|_0^{\pi} - \int_0^{\pi} \cancel{2}e^{2x}\frac{\cos 2x}{\cancel{2}}\,dx.$$

$$= \left[\frac{e^{2\pi}}{2} - \frac{1}{2}\right] - \int_0^{\pi} e^{2x}\cos 2x\,dx$$

$$\therefore \int_0^{\pi} e^{2x}\cos 2x\,dx = \frac{1}{2}\left[\frac{1}{2}e^{2\pi} - \frac{1}{2}\right]$$

$$= \frac{1}{4}\left[e^{2\pi} - 1\right]$$

30 Ans:

$$\int \frac{x\,dx}{\sqrt{1+x^2}}$$

$u = 1 + x^2$ $\quad x\,dx = \frac{1}{2}\,du$

$du = 2x\,dx$

$$= \int \frac{1}{2} \frac{du}{\sqrt{u}}$$

$$= \frac{1}{2} \int u^{-1/2}\,du = \frac{\cancel{1}}{\cancel{2}} \frac{u^{1/2}}{\cancel{1/2}} = \sqrt{u} + C$$

$$= \sqrt{1 + x^2} + C$$

32 $$\int \frac{x^3\,dx}{\sqrt{x^2+1}} =$$ $\quad u = 1 + x^2$

$$= \int \frac{x^2\,x\,dx}{\sqrt{x^2+1}}$$

$$= \int \frac{(u-1)\,\frac{1}{2}\,du}{\sqrt{u}} = \frac{1}{2} \int \sqrt{u} - u^{-1/2}\,du$$

$$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right] = \frac{u^{3/2}}{3} - \frac{u^{1/2}}{1}$$

$$= (u-1)\,u^{1/2} = x^2\sqrt{1+x^2} + C$$

(33) Ans!

$$\int \ln x \, x^n \, dx$$

$$= \ln x \frac{x^{n+1}}{n+1} - \int \frac{1}{\cancel{x}} \frac{\cancel{x^{n+1}}^{\,x^n}}{n+1} dx$$

$$= \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{n+1} \int x^n \, dx$$

$$= \frac{1}{n+1} \left[x^{n+1} \ln x - \frac{x^{n+1}}{n+1} \right]$$

$$= \frac{x^{n+1}}{n+1} \left[\ln x - \frac{1}{n+1} \right]$$

(52) $\int \frac{2x-1}{x^2} e^{2x} \, dx =$

$$\int -\frac{1}{x^2} e^{2x} \, dx = e^{2x} \frac{1}{x} - \int 2e^{2x} \frac{1}{x} dx$$

$$\Rightarrow \left(\frac{2}{x} - \frac{1}{x^2} \right) e^{2x} \, dx = \frac{e^{2x}}{x}.$$

Sec 7.3

5 Ans:

$\int \cos^3 x \, dx$

$du = \cos x \, dx$

$u = \sin x$

$= \int \cos^2 x \cos x \, dx$

$= \int (1 - \sin^2 x) \cos x \, dx$

$= \int (1 - u^2) \, du = u - \frac{u^3}{3} = \sin x - \frac{\sin^3 x}{3}$

8 $\int \sin^3 x \cos^3 x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int \sin^3 x \cos^2 x$
$\cos x \, dx$

$= \int u^3 (1 - u^2) \, du = \int u^3 - u^5 \, du = \frac{u^4}{4} - \frac{u^6}{6}$

$= \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6}.$

13 Ans:

$$\int \sin^2 x \cos^2 x \, dx$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\boxed{\cos^2 x = \frac{1+\cos 2x}{2}}$$

$$= \int \frac{1}{4}(1-\cos 2x)(1+\cos 2x)\, dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

$$= \frac{1}{4}\int \left[1-\cos^2 2x\right] dx$$

$$= \frac{1}{4}\int \sin^2 2x \, dx$$

$$= \frac{1}{4}\int \frac{1-\cos 4x}{2}\, dx = \frac{1}{8}\int \left[1-\cos 4x\right] dx.$$

$$= \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right]$$

22 $\int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx$

$$= \int \left[1+\tan^2 x\right] \sec^2 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$= \int (1+u^2)\, du = u + u^3/3$$

$$= \tan x + \frac{1}{3}\tan^3 x.$$

34 Ans.

$$\int \frac{dx}{\sqrt{9-x^2}}.$$

$$x = 3\cos\theta \qquad dx = -3\sin\theta\, d\theta$$

$$x^2 = 9\cos^2\theta$$

$$9 - x^2 = 9\sin^2\theta$$

$$= \int \frac{-\cancel{3}\,\cancel{\sin\theta}\, d\theta}{\cancel{3}\,\cancel{\sin\theta}} = \int -d\theta = -\theta$$

$$= -\cos^{-1}\frac{x}{3}$$

36 $\int \sqrt{9+x^2}\, dx = .$

$$x = 3\tan\theta$$

$$x^2 = 9\tan^2\theta$$

$$\sqrt{9+x^2} = \sqrt{9\sec^2\theta} = 3\sec\theta$$

$$dx = 3\sec^2\theta\, d\theta$$

$$= \int 3\sec\theta\; 3\sec^2\theta\, d\theta = \int 9\sec^3\theta\, d\theta.$$

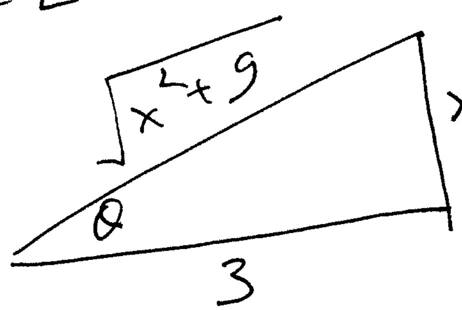
$$\int \sec^3 \theta \, d\theta =$$

$$\frac{\sec\theta \tan\theta}{2} + \frac{1}{2}\int \sec\theta \, d\theta .$$

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln\left|(\sec\theta + \tan\theta)\right| + C$$

$$\therefore \int \sqrt{9 + x^2}\, dx = .$$

$$\frac{9}{2}\left[\sec\theta\tan\theta + \ln\left|(\sec\theta + \tan\theta)\right|\right] + C$$



$$\tan\theta = \frac{x}{3}$$

$$\sec\theta = \frac{\sqrt{x^2+9}}{3}.$$

$$= \frac{9}{2}\left[\frac{\sqrt{x^2+9}\; x}{9} + \ln\left|\frac{1}{3}\left(\sqrt{x^2+9} + x\right)\right|\right.$$

40 Ans:

$$\int \frac{dx}{x\sqrt{7x^2-4}} = \int \frac{1}{2}\, d\theta = \frac{1}{2}\theta + C$$

$$= \frac{1}{2}\sec^{-1}\frac{\sqrt{7}x}{2}$$

$$= \frac{1}{\sqrt{7}} \int \frac{1}{x\sqrt{x^2-\frac{4}{7}}}\, dx.$$

$$\frac{dx}{x} = \tan\theta\, d\theta.$$

$$\sqrt{7x^2-4} = 2\tan\theta$$

$$x = \frac{2}{\sqrt{7}}\sec\theta$$

$$x^2 = \frac{4}{7}\sec^2\theta$$

$$\sqrt{x^2-\frac{4}{7}} = \sqrt{\frac{4}{7}(\sec^2\theta - 1)}$$

$$= \sqrt{\frac{4}{7}\tan^2\theta} = \frac{2}{\sqrt{7}}\frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{\sqrt{7}} \int \frac{\sqrt{7}}{2} \frac{\cos\theta}{\frac{\sqrt{4}}{\sqrt{7}}\sin\theta} \cos\theta$$

$$= \frac{1}{\cancel{\sqrt{7}}}\,\frac{\cancel{7}}{4} \int \frac{\cos^2\theta}{\sin\theta}\,\frac{2}{\cancel{\sqrt{7}}}\, \cancel{\sec\theta \tan\theta}\, d\theta$$

$$\frac{\cos^2\theta}{\sin\theta}\,\frac{1}{\cos\theta}\,\frac{\sin\theta}{\cos\theta}.$$

$$= \frac{1}{2}\theta$$

$$= \frac{1}{2}\sec^{-1}\frac{\sqrt{7}x}{2}$$

47

$$\int \frac{dx}{\sqrt{x^2-2x+6}}.$$

$$\sqrt{x^2-2x+6} = \sqrt{(x-1)^2+5}$$

$u = x-1$

$du = dx$

$$= \int \frac{du}{\sqrt{u^2+5}}$$

$$= \ln\left|\sqrt{5+u^2} + u\right| + C$$

$$= \ln\left|\sqrt{5+x^2-2x+1} + x-1\right| + C$$

$$= \ln\left|\sqrt{x^2-2x+6} + x-1\right| + C$$