H. W. 4 (Answers)

(1) $\displaystyle\int \frac{2x+5}{\sqrt{x^2+5x}}\,dx = I$

$u = x^2+5x$

$du = (2x+5)\,dx$

$$I = \int \frac{du}{\sqrt{u}} = \int u^{-1/2}\,du.$$

$$= \frac{u^{-1/2+1}}{-\frac{1}{2}+1} = \frac{u^{1/2}}{1/2} = 2\sqrt{u}.$$

$$= 2\sqrt{x^2+5x}$$

(2) $\displaystyle\int \frac{\ln x}{x}\,dx = I$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$I = \int u\,du = \frac{u^2}{2} = \frac{(\ln x)^2}{2}$$

(4) $\displaystyle\int \cos x\, e^{\sin x}\,dx = I$

$u = \sin x$

$du = \cos x\,dx$

$$I = \int e^u\,du = e^u = e^{\sin x}.$$

(7) $\int (1+\cot x)^4 \operatorname{cosec}^2 x \, dx = I$

$u = 1 + \cot x$

$\frac{du}{dx} = -\operatorname{cosec}^2 x$

$\therefore \; I = \int u^4 (-du) = -\int u^4 \, du.$

$= -\frac{u^5}{5}$

$= -\frac{(1+\cot x)^5}{5}$

(12) $\int \frac{2x-1}{(4x^2-4x)^2} \, dx = I$

$u = 4x^2 - 4x$

$du = (8x-4)\,dx = 4(2x-1)\,dx$

$I = \int \frac{\frac{1}{4}\,du}{u^2} = \frac{1}{4}\int u^{-2}\,du$

$= \frac{1}{4}\,\frac{u^{-2+1}}{-2+1} = \frac{1}{4}\,\frac{u^{-1}}{-1} = -\frac{1}{4}\,\frac{1}{u}$

$= -\frac{1}{4}\,\frac{1}{4x^2-4x}$

$$I = -\frac{1}{4}\,\frac{1}{u} = -\frac{1}{4}\,\frac{1}{4x^2-4x}$$

$$= \frac{1}{4}\,\frac{-1}{4x(x-1)}$$

$$I = \frac{1}{16x(1-x)}$$

$\cancel{13}$ (17) $\int x e^{ax}\,dx$.

Using formula 484 Appendix D we have

$$\int x e^{ax}\,dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$$

(18) $$\int \frac{dx}{a+be^{2x}} = \int \frac{e^{-2x}\,dx}{b+ae^{-2x}} = I$$

$$b+ae^{-2x} = u$$

$$du = a(-2)e^{-2x}\,dx = -2ae^{-2x}\,dx$$

$$-\frac{1}{2a}\,du = e^{-2x}\,dx$$

$$I = \int -\frac{1}{2a}\,\frac{du}{u} = -\frac{1}{2a}\ln|u| = -\frac{1}{2a}\ln\left|b+ae^{-2x}\right|$$

Alternatively, using 489 Appendix D we have

$$\int \frac{du}{p+qe^{au}} = \frac{u}{p} - \frac{1}{ap}\ln|p+qe^{au}|$$

$$\int \frac{dx}{a+be^{2x}} = \frac{x}{a} - \frac{1}{2a}\ln|a+be^{2x}|$$

$$= \frac{x}{a} - \frac{1}{2a}\ln\left[|e^{2x}||b+ae^{-2x}|\right]$$

$$= \frac{x}{a} - \frac{1}{2a}\left[2x + \ln|b+ae^{-2x}|\right]$$

$$= -\frac{1}{2a}\ln|b+ae^{-2x}|$$

(22) $$\int \frac{dx}{x\sqrt{1-9x^2}} = I$$

$$\sqrt{1-9x^2} = \sqrt{9\left(\frac{1}{9}-x^2\right)} = 3\sqrt{\left(\frac{1}{3}\right)^2 - x^2}$$

$$I = \frac{1}{3}\int \frac{dx}{x\sqrt{\left(\frac{1}{3}\right)^2 - x^2}}$$

$$= \frac{1}{\cancel{3}}(-\cancel{3})\ln\left|\frac{\frac{1}{3}+\sqrt{\frac{1}{9}-x^2}}{x}\right| = -\ln\left|\frac{\frac{1}{3}+\sqrt{\frac{1}{9}-x^2}}{x}\right|$$

$$I = \ln\left|\frac{x}{\frac{1}{3}+\sqrt{\frac{1}{9}-x^2}}\right|$$

28 $\int x\sqrt{1+x}\,dx = I$

$u = 1+x$

$du = dx$

$$I = \int (u-1)\sqrt{u}\,du$$

$$= \int (u\sqrt{u} - \sqrt{u})\,du$$

$$= \int (u^{3/2} - u^{1/2})\,du$$

$$= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}$$

$$I = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}$$

31 $\int \frac{dx}{1+e^{2x}} = \int \frac{e^{-2x}\,dx}{1+e^{-2x}} = I$

$u = e^{-2x} + 1$

$du = -2e^{-2x}\,dx.$

$\Rightarrow e^{-2x}\,dx = -\frac{1}{2}\,du.$

$I = \int -\frac{1}{2}\frac{du}{u} = -\frac{1}{2}\ln|u|.$

$= -\frac{1}{2}\ln|1+e^{-2x}|$

33 $\int \frac{x^3\,dx}{\sqrt{4x^4+1}} = I.$

$u = 4x^4 + 1$

$du = 16x^3\,dx \Rightarrow x^3\,dx = \frac{1}{16}\,du$

$I = \int \frac{1}{16}\frac{du}{\sqrt{u}} = \frac{1}{16}\frac{u^{-1/2+1}}{-\frac{1}{2}+1} = \frac{1}{16}\frac{\sqrt{u}}{1/2}$

$= \frac{1}{8}\sqrt{u} = \frac{1}{8}\sqrt{4x^4+1}$

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problem 1

$$\int xe^{-2x}\,dx$$

$$= x\frac{e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2}\,dx$$

$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$

$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\,\frac{e^{-2x}}{-2}$$

$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$$

(2) $\int x\sin x\,dx = I_1$

~~Let $I_2 = \int x\cos x\,dx$~~

$$I_1 = -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x$$

(9) $\int x^2 \ln x\,dx = \ln x\,\frac{x^3}{3} - \int \frac{1}{x}\,\frac{x^{3\,2}}{3}\,dx$

$$= \frac{x^3}{3}\ln x - \int \frac{x^2}{3} = \frac{x^3}{3}\ln x - \frac{1}{3}\,\frac{x^3}{3}$$

$\frac{x^3}{3}\ln x - \frac{1}{9}x^3$

(13)

$$\int \ln(1+x^2)\,dx = I$$

$$= \ln(1+x^2)\,x - \int \frac{1}{1+x^2}\,2x^2\,dx.$$

$$\int \frac{2x^2\,dx}{x^2+1} = \int 2 - \frac{2}{x^2+1}\,dx.$$

$$= 2x - \int \frac{2}{x^2+1}\,dx.$$

$$= 2x - 2\tan^{-1}x$$

$$I = x\ln(1+x^2) - 2x + 2\tan^{-1}x.$$