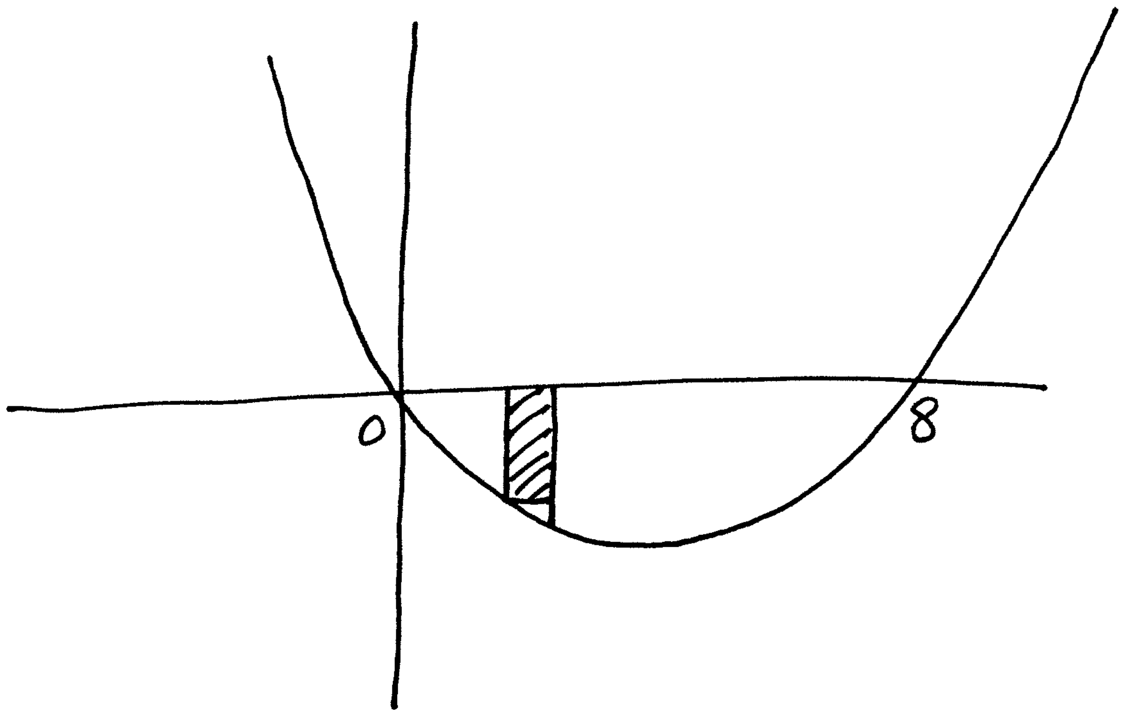
Math 1352 Calculus II

Home Work I (answers)

2. $y = x^2 - 8x = x(x-8)$

$y = 0$



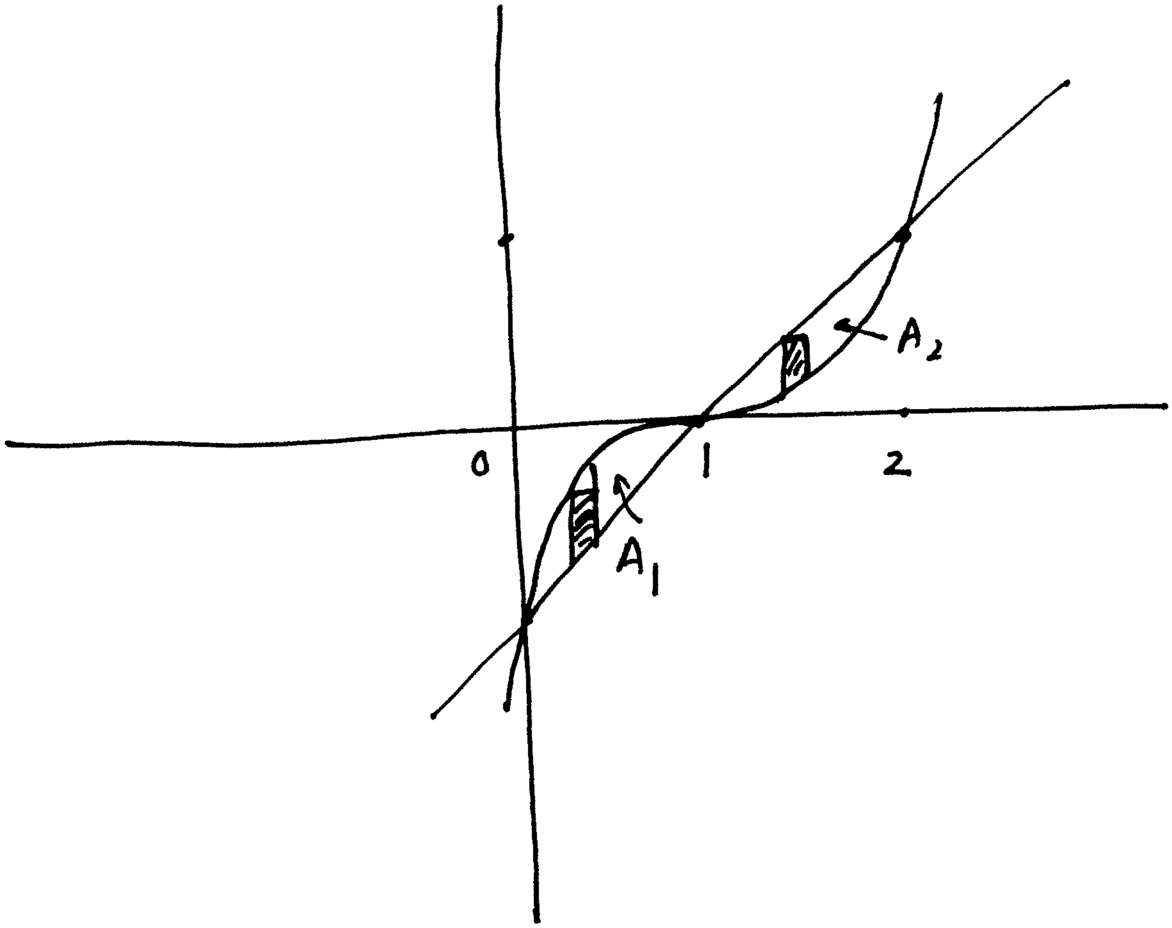
$$\text{Area} = \int_0^8 0 - (x^2 - 8x)\,dx$$

$$= -\frac{x^3}{3} + 8\frac{x^2}{2}\Big|_0^8$$

$$= -\frac{8^3}{3} + \frac{8^3}{2} = 8^3\,\frac{3-2}{6} = \frac{64 \cdot \cancel{8}^{4}}{\cancel{6}_{3}}$$

$$= \frac{256}{3}$$

④ ②



$$A_1 = \int_0^1 \left[(x-1)^3 - (x-1)\right] dx$$

$$A_2 = \int_1^2 \left[(x-1) - (x-1)^3\right] dx$$

$$(x-1)^3 = x^3 - 3x^2 + 3x - 1$$

$$(x-1)^3 - (x-1) = x^3 - 3x^2 + 3x - \cancel{1} - x + \cancel{1}$$

$$= x^3 - 3x^2 + 2x$$

$A_1 =$

$$\int_0^1 \left[x^3 - 3x^2 + 2x\right] dx$$

$$= \frac{x^4}{4} - \cancel{3}\frac{x^3}{\cancel{3}} + \cancel{2}\frac{x^2}{\cancel{2}} \Big|_0^1$$

$$= \frac{1}{4} - 1 + 1 = \frac{1}{4}$$

$$A_2 = \int_1^2 \left[-x^3 + 3x^2 - 2x\right] dx$$

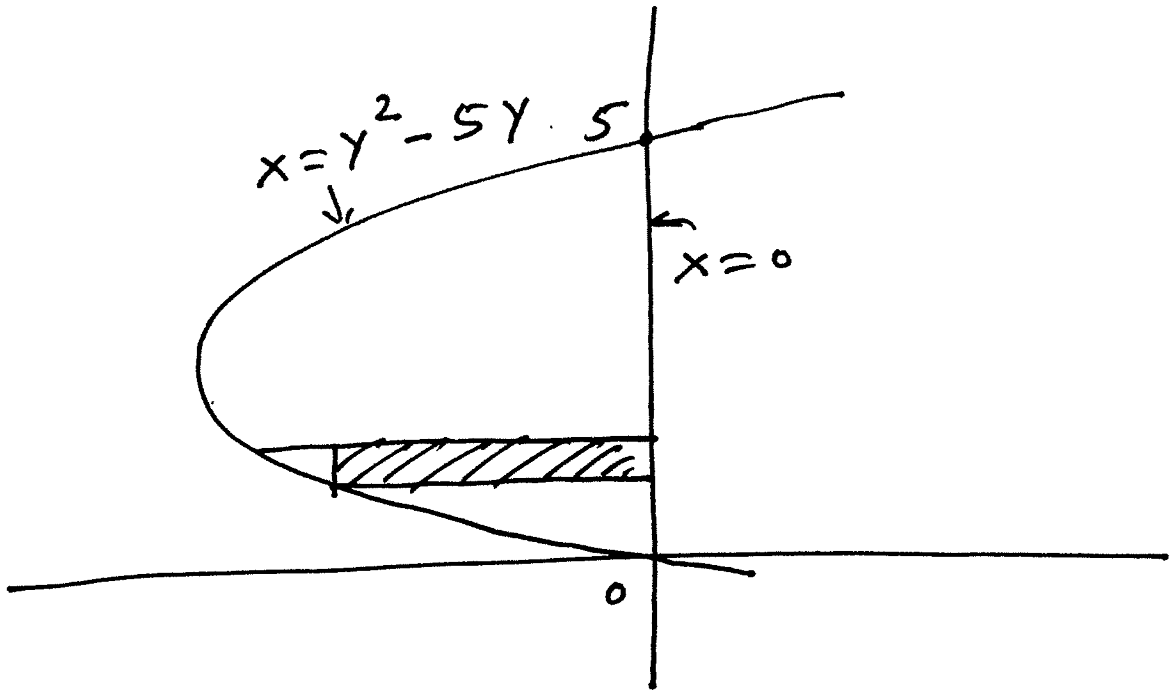
$$= -\frac{x^4}{4} + \cancel{3}\frac{x^3}{\cancel{3}} - \cancel{2}\frac{x^2}{\cancel{2}} \Big|_1^2$$

$$= \left[-\frac{16}{4} + 8 - 4\right] - \left[-\frac{1}{4} + \cancel{1} - \cancel{1}\right]$$

$$= -\frac{16}{4} + 4 + \frac{1}{4} = \frac{-16 + 16 + 1}{4} = \frac{1}{4}$$

$$A = A_1 + A_2 = \frac{1}{2}$$

5



$$\text{Area} = \int_0^5 \left[0 - (y^2 - 5y)\right] dy$$

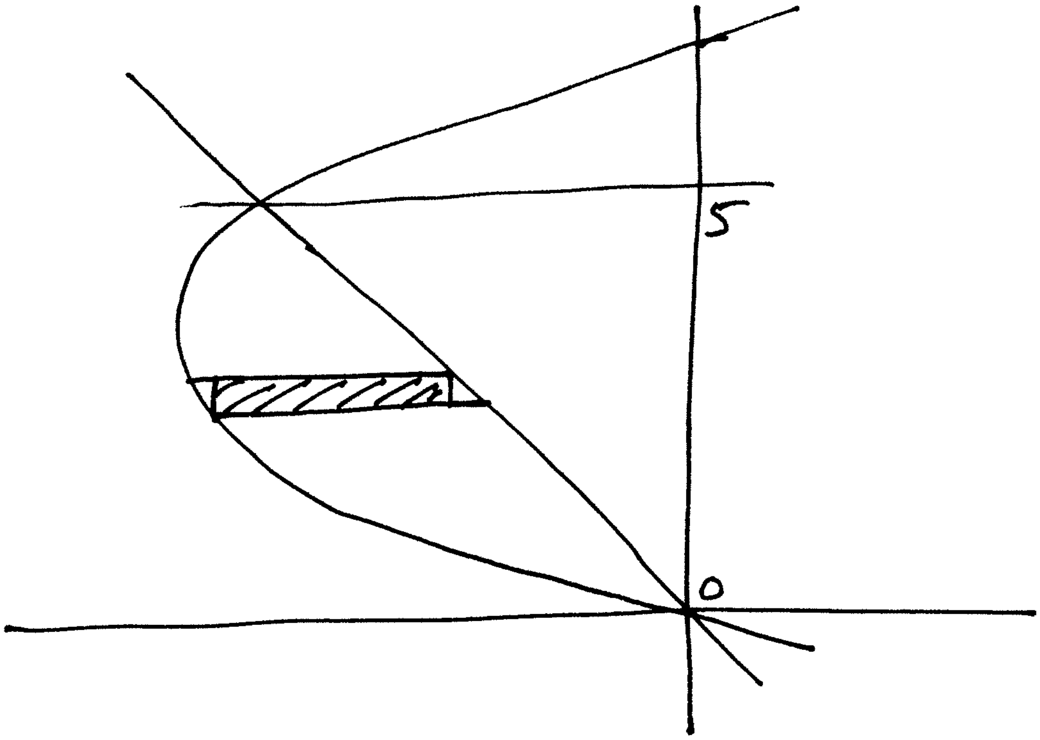
$$= -\frac{y^3}{3} + 5\frac{y^2}{2} \Big|_0^5$$

$$= -\frac{125}{3} + \frac{5}{2} \cdot 25$$

$$= \frac{125}{2} - \frac{125}{3} = 125 \quad \frac{1}{6}$$

$$= \frac{125}{6}.$$

6



$$\text{Area} = \int_0^5 \left[(-y) - (y^2 - 6y)\right] dy$$

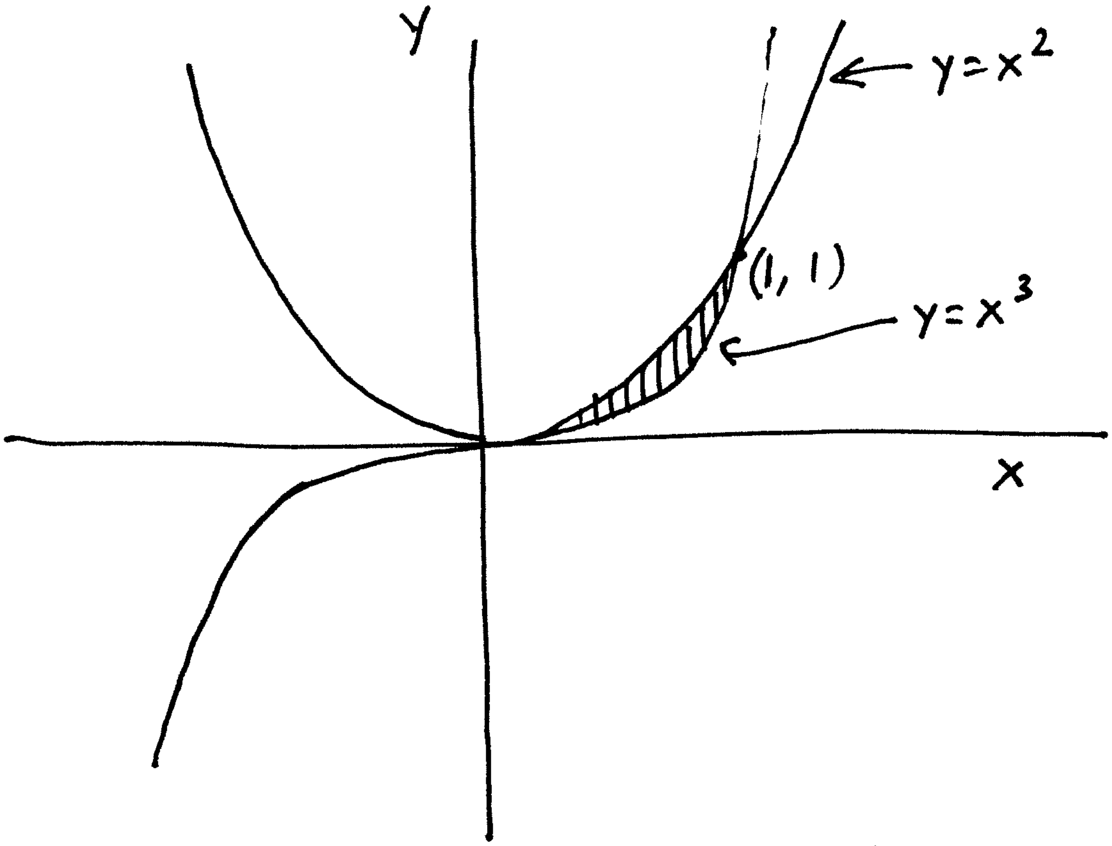
$$= \int_0^5 \left[-y - y^2 + 6y\right] dy$$

$$= \int_0^5 \left[-y^2 + 5y\right] dy$$

$$= -\frac{y^3}{3} + 5\,\frac{y^2}{2}\Bigg|_0^5$$

$$= -\frac{125}{3} + \frac{5}{2}\cdot 25 = 125\,\frac{1}{6} = 125/6.$$

Problem 9



To find the intersection of two curves

$y=x^2$ and $y=x^3$

we equate

$$x^2 = x^3 \Rightarrow x^2 - x^3 = 0$$
$$\Rightarrow x^2(x-1) = 0$$
$$\Rightarrow x=0,\ x=1.$$

$\therefore$ (0, 0) and (1, 1) are the two points of intersection.

Using vertical strip we have

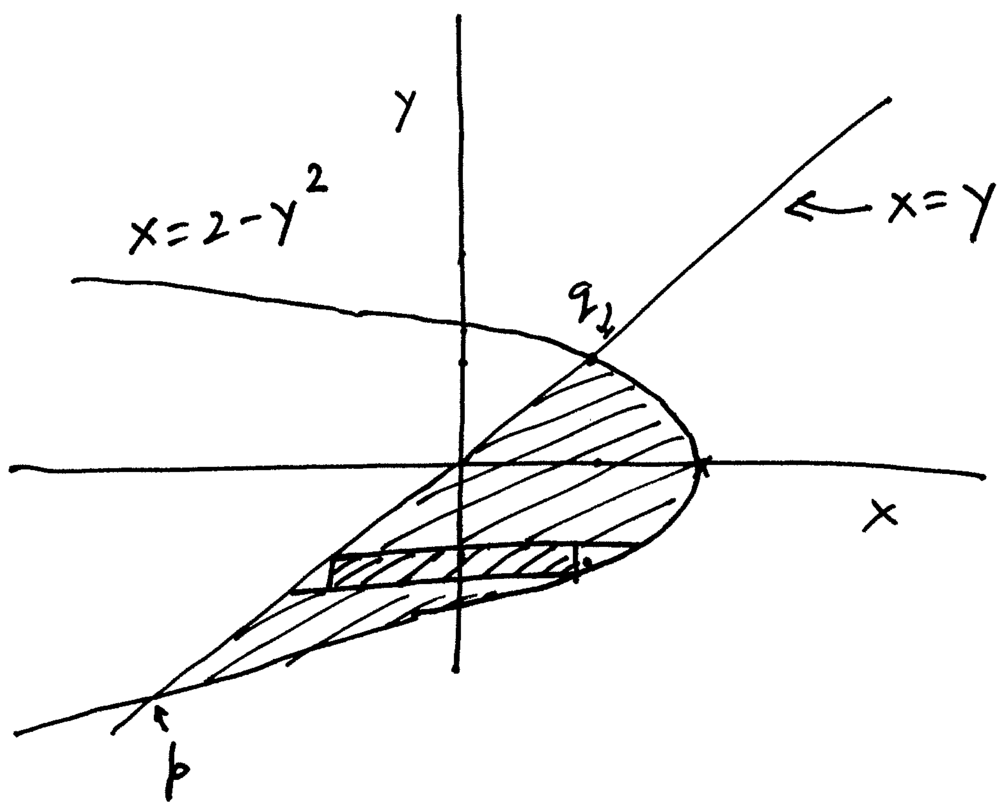
$$\text{Area} = \int_0^1 \left[x^2 - x^3\right] dx$$

$$= \frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$$

— x —

Problem 15



The point of intersections are calculated as follows:

$$y = 2 - y^2$$

$$\Rightarrow y^2 + y - 2 = 0$$

$$\Rightarrow y = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$= \frac{-1 \pm 3}{2} = \frac{-1-3}{2}, \frac{-1+3}{2}$$

$$= -2, 1$$

$\therefore$ p is $(-2, -2)$

q is $(1, 1)$

Using horizontal strips we have

$$\text{Area} = \int_{-2}^{1} \left[2 - y^2 - y\right] dy$$

$$= 2y - \frac{y^3}{3} - \frac{y^2}{2} \Big|_{-2}^{1}$$

9

$$\left[2-\frac{1}{3}-\frac{1}{2}\right]-\left[2(-2)-\frac{(-2)^3}{3}-\frac{(-2)^2}{2}\right]$$

$$=\left(2-\frac{1}{3}-\frac{1}{2}\right)-\left(-4+\frac{8}{3}-2\right)$$

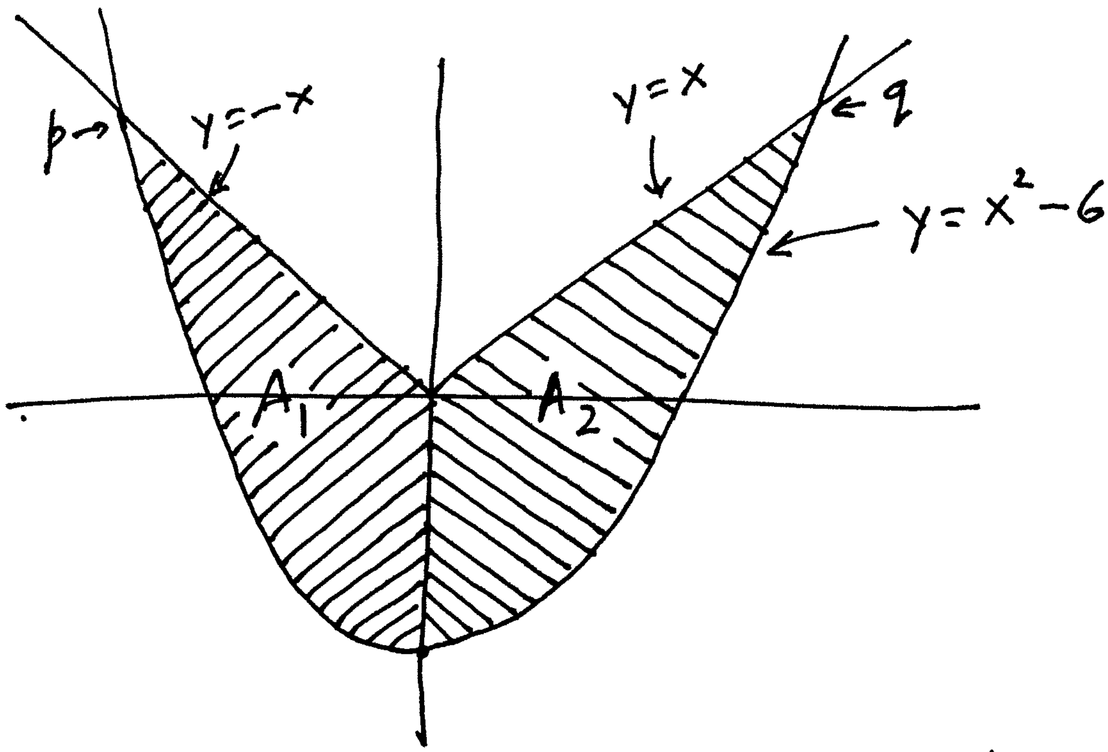
$$=2-\frac{1}{3}-\frac{1}{2}+4-\frac{8}{3}+2$$

$$=8-\frac{1}{3}-\frac{1}{2}-\frac{8}{3}$$

$$=8-\frac{2+3+16}{6}.$$

$$=8-\frac{\cancel{21}\,7}{\cancel{6}\,2}=\frac{16-7}{2}=\frac{9}{2}=4.5.$$

Problem 20



To obtain the point of intersection 'p' we equate

$$-x = x^2 - 6$$

$$\Rightarrow x^2 + x - 6 = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 24}}{2} = \frac{-1 \pm \sqrt{25}}{2}$$

$$= \frac{-1 \pm 5}{2} = \frac{-1-5}{2}, \frac{-1+5}{2}$$

$$= -3, 2$$

The x-co-ordinate of p is negative.

Hence $x = -3$, therefore $y = 3$. p is $(-3, 3)$

To obtain the point of intersection 'q' we equate

$$x = x^2 - 6$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1 + 24}}{2} = \frac{1 \pm 5}{2}$$

$$= 3, -2$$

The x coordinate of q is positive

Hence $x = 3$, therefore $y = 3$.

q is $(3, 3)$.

Let us use vertical strips.

Area of A_1 =

$$\int_{-3}^{0} (-x) - (x^2 - 6)\, dx$$

$$= \int_{-3}^{0} (-x - x^2 + 6)\, dx$$

$$= -\frac{x^2}{2} - \frac{x^3}{3} + 6x \Big|_{-3}^{0}$$

$$= 0 - \left[-\frac{(-3)^2}{2} - \frac{(-3)^3}{3} + 6(-3) \right]$$

$$= \left[\frac{(-3)^2}{2} + \frac{(-3)^3}{3} - 6(-3) \right]$$

$$= \frac{9}{2} - \frac{27}{3} + 18$$

$$= \frac{27 - 54 + 108}{6} = \frac{81}{6} = \frac{27}{2}$$

$$= 13\tfrac{1}{2}.$$

Area of A_2 =

$$\int_0^3 [x - (x^2 - 6)]\,dx = 13\tfrac{1}{2}$$

Total area of A_1 & A_2 = 27.

Problem 22

$y = x^3 - 2x^2 - x + 2$

$x = 1$ is one of the root of the polynomial

$x^3 - 2x^2 - x + 2$.

Hence

$(x^3 - 2x^2 - x + 2) = (x-1)$ [a quadratic polynomial]

$$
\begin{array}{r l}
x-1\,) & \cancel{x^3} - 2x^2 - x + 2 \;(\; x^2 - x - 2 \\
& \underline{\cancel{x^3} \mp x^2} \\
& - \cancel{x^2} - x + 2 \\
& \underline{- \cancel{x^2} \pm x} \\
& - 2x + 2 \\
& \underline{- 2x + 2} \\
\end{array}
$$

$\therefore \; x^3 - 2x^2 - x + 2 = (x-1)(x^2 - x - 2)$

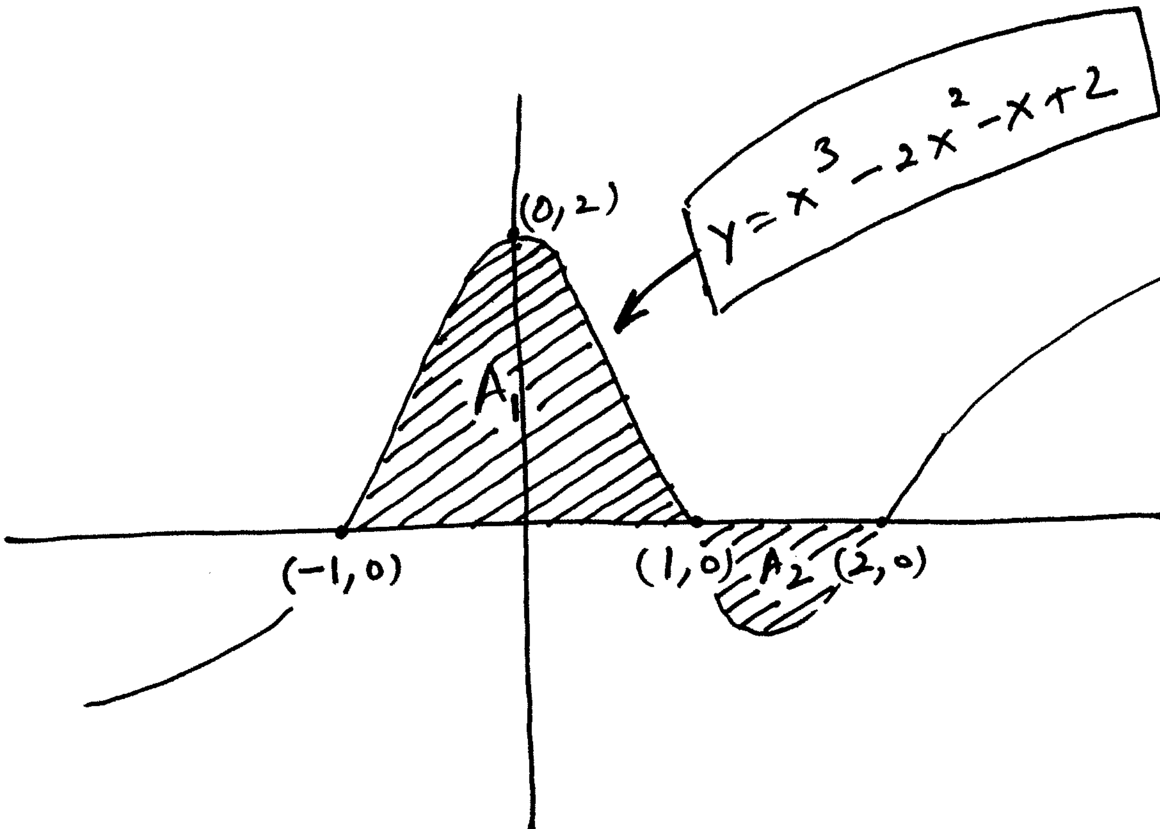
To find roots of $x^2 - x - 2$

$x^2 - x - 2 = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1 + 8}}{2}$

$= \dfrac{1 \pm 3}{2} = 2, -1$

$\therefore$ $x^3 - 2x^2 - x + 2 =$

$$(x-1)(x+1)(x-2).$$



using vertical strips.

Area of A_1 =

$$\int_{-1}^{1} \left[x^3 - 2x^2 - x + 2\right] dx$$

$$\left[\frac{x^4}{4} - 2\frac{x^3}{3} - \frac{x^2}{2} + 2x\right]_{-1}^{1}$$

$$= \left[\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right] - \left[\frac{1}{4} + \frac{2}{3} - \frac{1}{2} - 2\right]$$

$$= -\frac{2}{3} + 2 - \frac{2}{3} + 2.$$

$$= 4 - \frac{4}{3}. = \frac{12-4}{3} = \frac{8}{3}.$$

Area of A_2 =

$$\int_1^2 \left[-x^3 + 2x^2 + x - 2\right] dx$$

$$= -\frac{x^4}{4} + 2\frac{x^3}{3} + \frac{x^2}{2} - 2x \Big|_1^2$$

$$= \left[-4 + \frac{2}{3}\cdot 8 + 2 - 4\right] - \left[-\frac{1}{4} + \frac{2}{3} + \frac{1}{2} - 2\right]$$

$$\left(-6+\frac{16}{3}\right)+\left(\frac{1}{4}-\frac{2}{3}-\frac{1}{2}+2\right)$$

$$=\frac{-72+64+3-8-6+24}{12}$$

$$=\frac{91-86}{12}=\frac{5}{12}$$

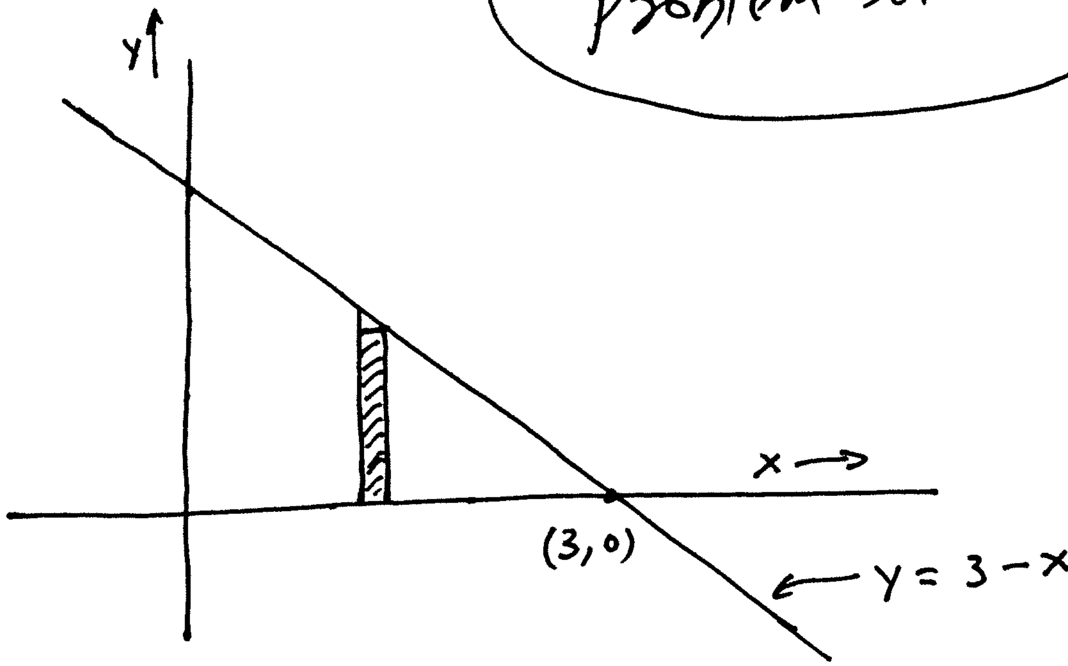
Total Area =

$$\frac{8}{3}+\frac{5}{12}=\frac{32+5}{12}$$

$$=37/12.$$

Problem 1

Page 372
Problem set 6.2



Area of the cross section at x is

$(3-x)^2$

$$\text{Volume} = \int_0^3 (3-x)^2 \, dx$$

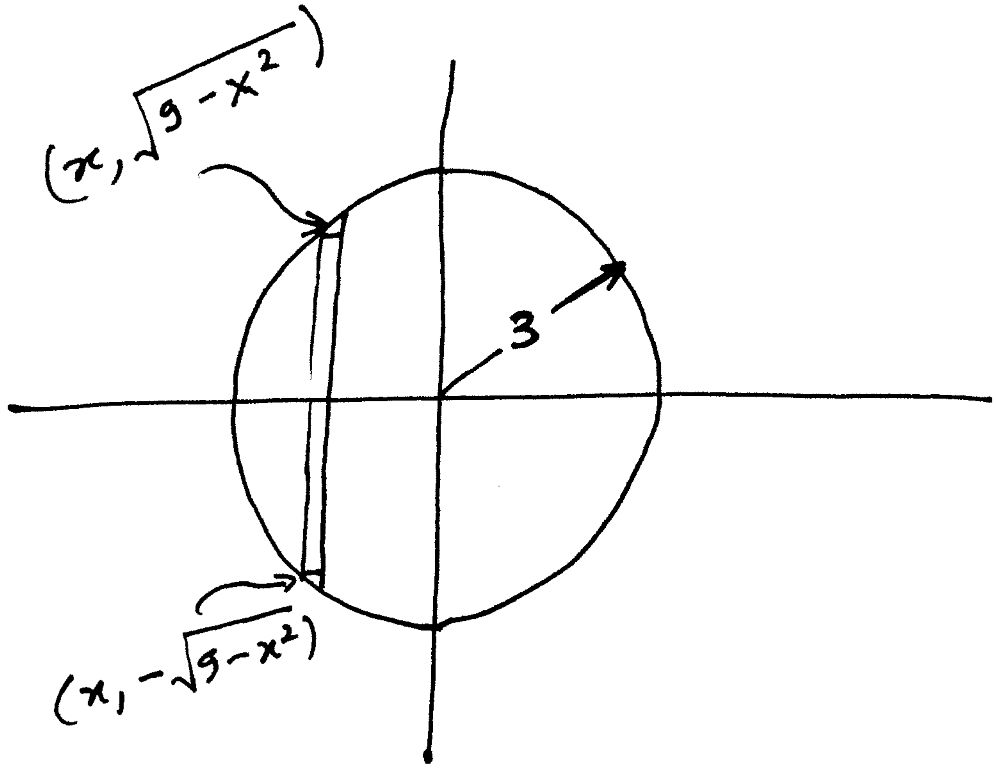
$$= \int_0^3 (9 - 6x + x^2) \, dx$$

$$= 9x - \frac{6x^2}{2} + \frac{x^3}{3} \Big|_0^3$$

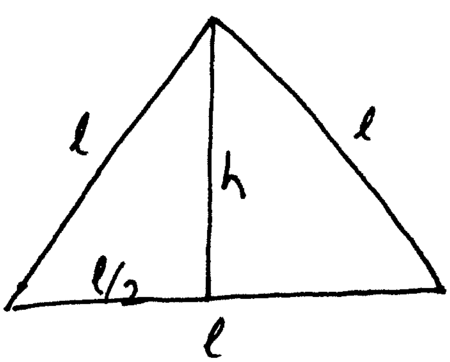
$$= 27 - \frac{54}{2} + \frac{27}{3} = \frac{162 - 162 + 54}{6}$$

$$= 54/6 = \frac{27}{3} = 9.$$

Problem 5



The base of the equilateral triangle at x is of length $2\sqrt{9-x^2}$.



$\therefore l = 2\sqrt{9-x^2}$.

$$l^2 = h^2 + l^2/4.$$

$$h^2 = l^2\left(1-\frac{1}{4}\right) = l^2\,\frac{3}{4}.$$

$$h = l\,\frac{\sqrt{3}}{2}.$$

$$\text{Area} = \frac{l}{2}\,l\,\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}\,l^2.$$

$$= \frac{\sqrt{3}}{4}\,4\,(9-x^2) = \sqrt{3}\,(9-x^2)$$

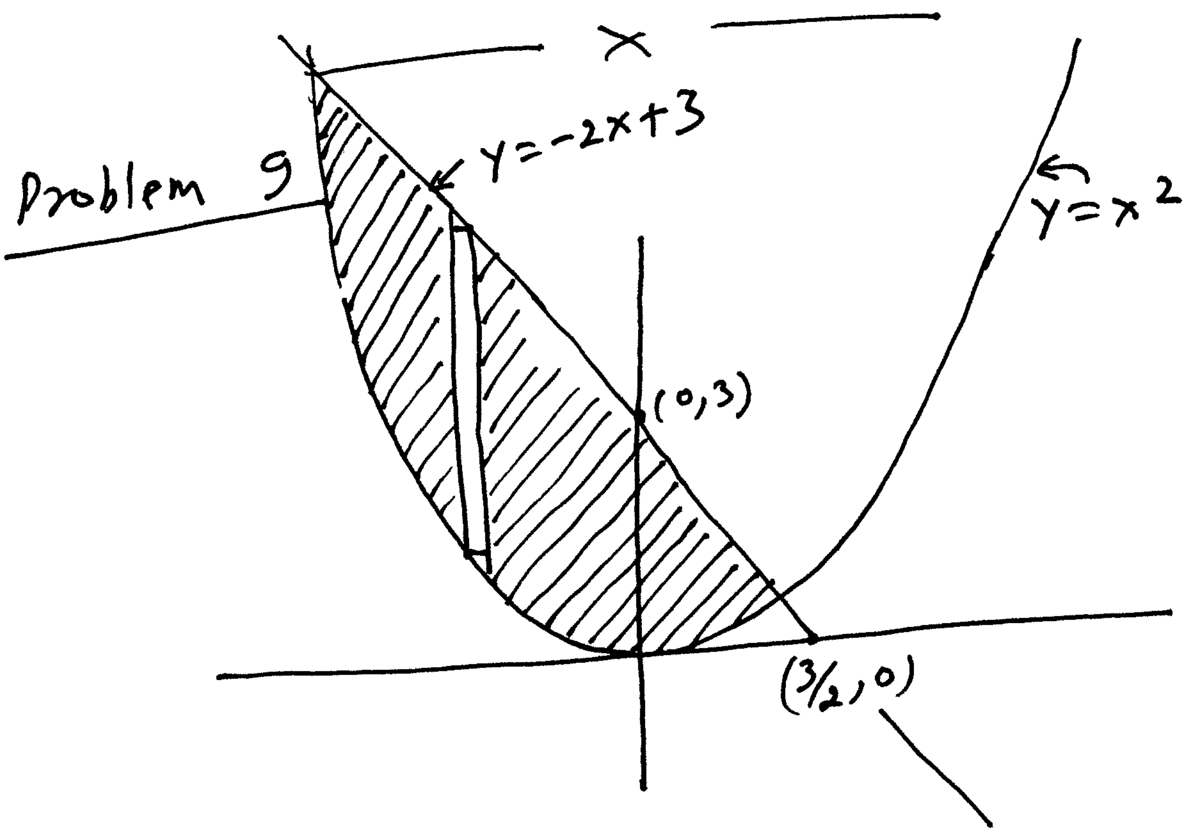
Volume =

$$\int_{-3}^{3} \sqrt{3}\,(9-x^2)\,dx.$$

$$= \sqrt{3}\left(9x - \frac{x^3}{3}\right)\Big|_{-3}^{3}$$

$$= \sqrt{3}\,\big[(27-9) - (-27+9)\big]$$

$$= 36\sqrt{3}$$



The points of intersection are

$$2x + x^2 - 3 = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{4 + 12}}{2}$$

$$= \frac{-2 \pm 4}{2} = \frac{-2+4}{2}, \frac{-2-4}{2}$$

$$= 1, -3.$$

when $x = 1$, $y = 1$.

when $x = -3$, $y = 9$

The cross section at x is a semicircle with base length: $(-2x+3) - x^2$

$$D = -x^2 - 2x + 3.$$

Area of the cross section =

$$\frac{1}{2}\pi r^2 = \frac{1}{2}\pi \frac{D^2}{4} = \frac{1}{8}\pi D^2 = \frac{1}{8}\pi\left[-x^2-2x+3\right]^2$$

Volume

$$\int_{-3}^{1} \frac{1}{8}\pi \left[-x^2-2x+3\right]^2 dx.$$

$$\begin{array}{r} -x^2-2x+3 \\ -x^2-2x+3 \\ \hline x^4+2x^3-3x^2 \qquad\qquad \\ +2x^3+4x^2-6x \qquad \\ -3x^2-6x+9. \\ \hline x^4+4x^3-2x^2-12x+9. \end{array}$$

$p(x) = \frac{x^5}{5} + x^4 - \frac{2}{3}x^3 - 6x^2 + 9x$

$$\int \left[x^4+4x^3-2x^2-12x+9\right] dx$$

$$= \frac{x^5}{5} + \cancel{4}\frac{x^4}{\cancel{4}} - 2\frac{x^3}{3} - \overset{6}{\cancel{12}}\frac{x^2}{\cancel{2}} + 9x = p(x).$$

$$\text{Volume} = \frac{1}{8}\pi\left[p(1) - p(-3)\right].$$