

Financial Time Series

Lecture 1: Introduction

Financial time series (FTS) analysis is concerned with theory and practice of asset valuation over time.

What is the difference, if any, from traditional time series analysis?

Two topics are highly related, but FTS has added uncertainty, because it must deal with the ever-changing business & economic environment and the fact that **volatility** is not directly observed.

Objective of the course

- to access financial data online and to process the embedded information
- to provide basic knowledge of FTS data such as skewness, heavy tails, and measure of dependence between asset returns
- to introduce statistical tools & econometric models useful for analyzing these series.
- to gain experience in analyzing FTS
- to introduce recent developments in financial econometrics and their applications, e.g., high-frequency finance
- to study methods for assessing market risk, credit risk, and expected loss. The methods discussed include Value at Risk, expected shortfall, and tail dependence.

- to analyze high-dimensional asset returns, including co-movement

Examples of financial time series

1. Daily log returns of Apple stock: 2007 to 2016 (10 years). Data downloaded using `quantmod`
2. The VIX index
3. CDS spreads: Daily 3-year CDS spreads of JP Morgan from July 20, 2004 to September 19, 2014.
4. Quarterly earnings of Coca-Cola Company: 1983-2009
Seasonal time series useful in
 - earning forecasts
 - pricing weather related derivatives (e.g. energy)
 - modeling intraday behavior of asset returns
5. US monthly interest rates (3m & 6m Treasury bills)
Relations between the two asset returns? Term structure of interest rates
6. Exchange rate between US Dollar vs Euro
Fixed income, hedging, carry trade
7. Size of insurance claims
Values of fire insurance claims ($\times 1000$ Krone) that exceeded 500 from 1972 to 1992.
8. High-frequency financial data:
Tick-by-tick data of Caterpillars stock: January 04, 2010.

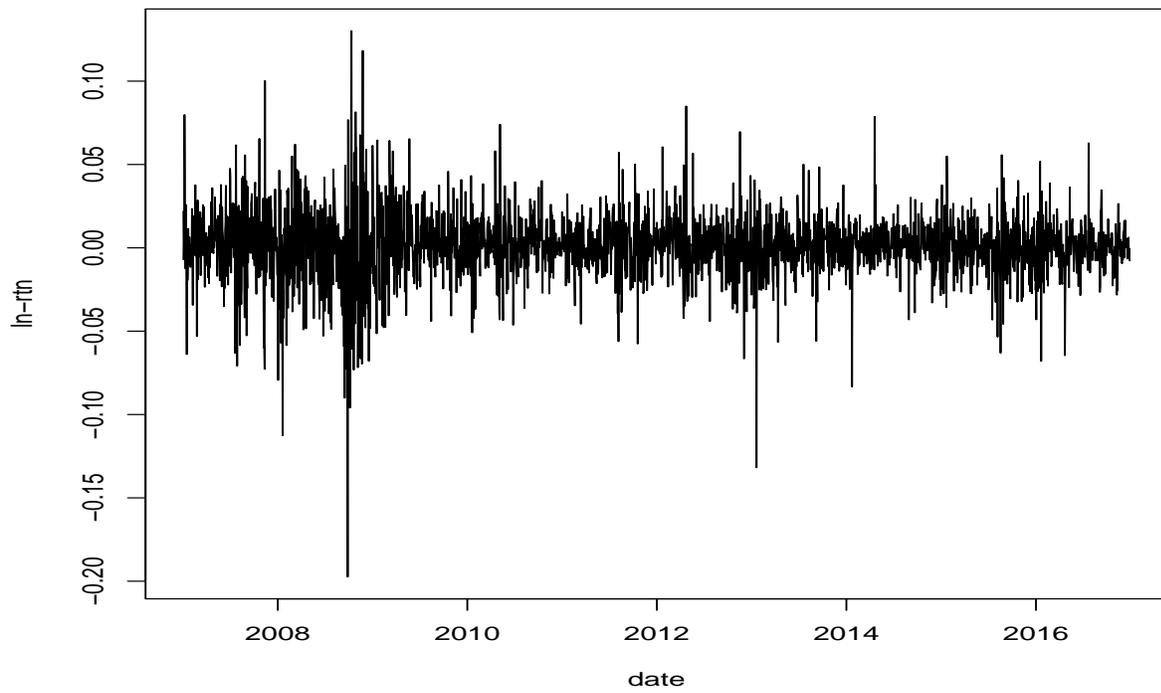


Figure 1: Daily log returns of Apple stock from 2007 to 2016

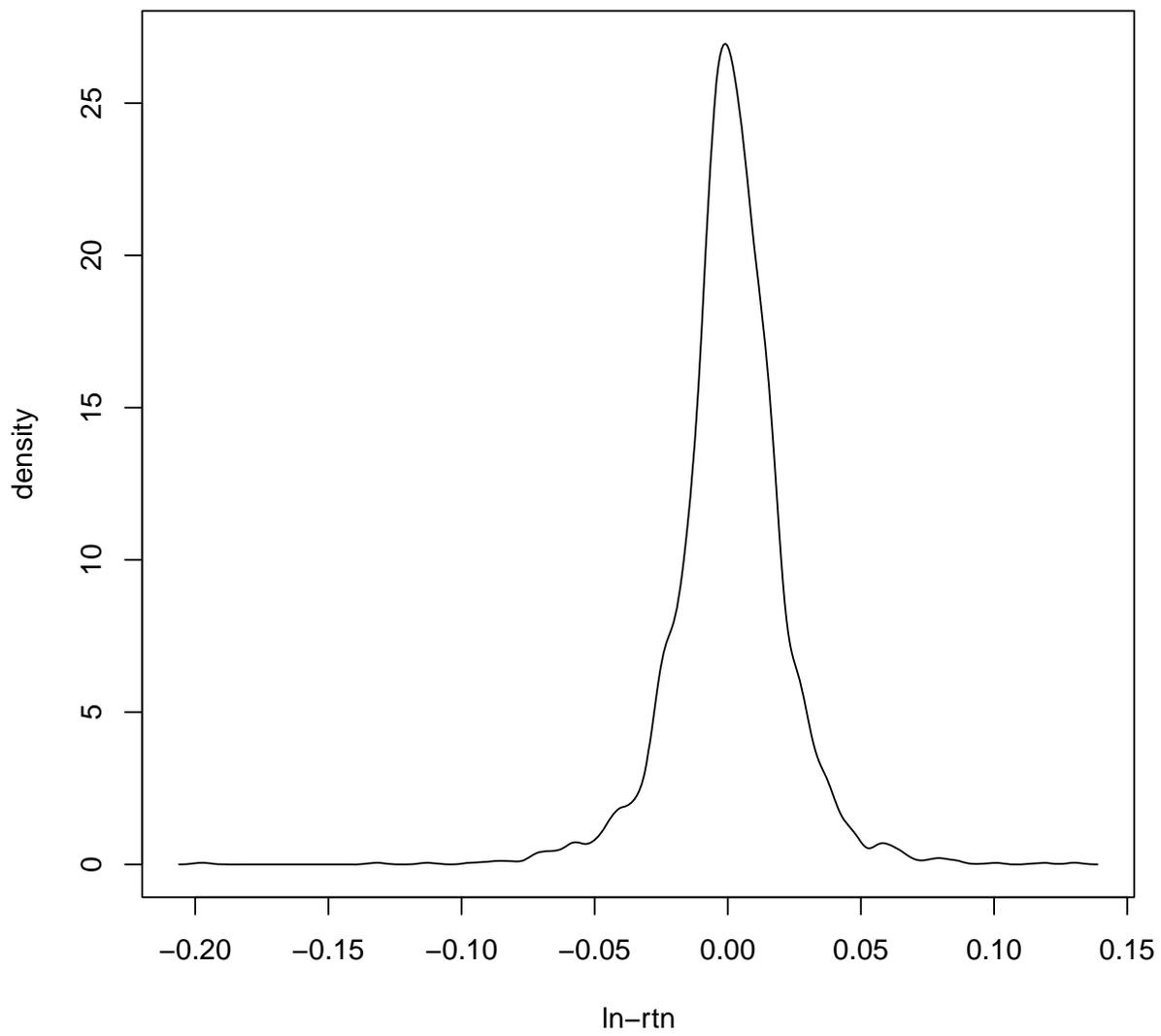


Figure 2: Empirical density function of daily log returns of Apple stock: 2007 to 2016

CDS of JPM: 3-yr spread

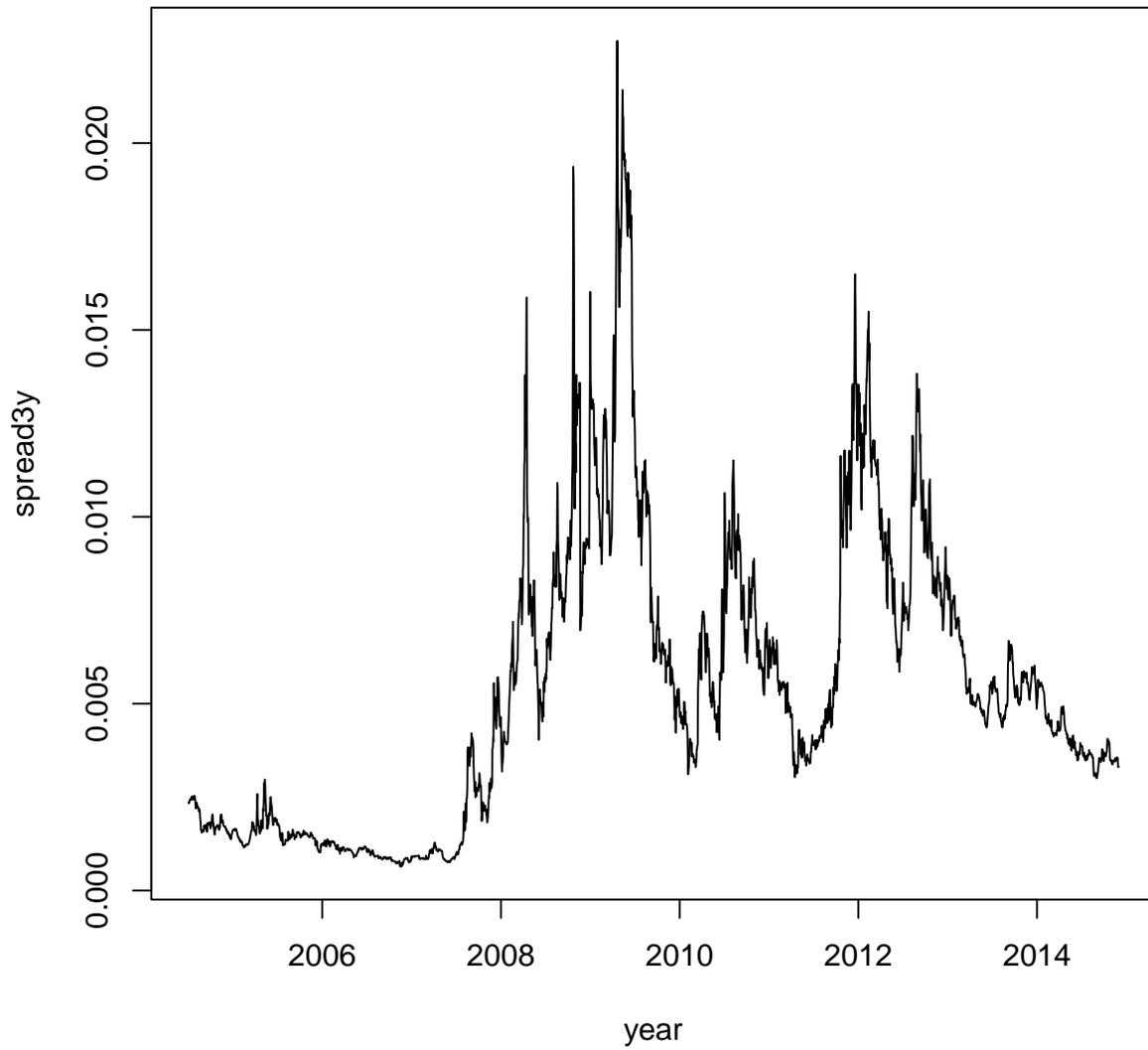


Figure 3: Time plot of daily 3-year CDS spreads of JPM: from July 20, 2004 to September 19, 2014.

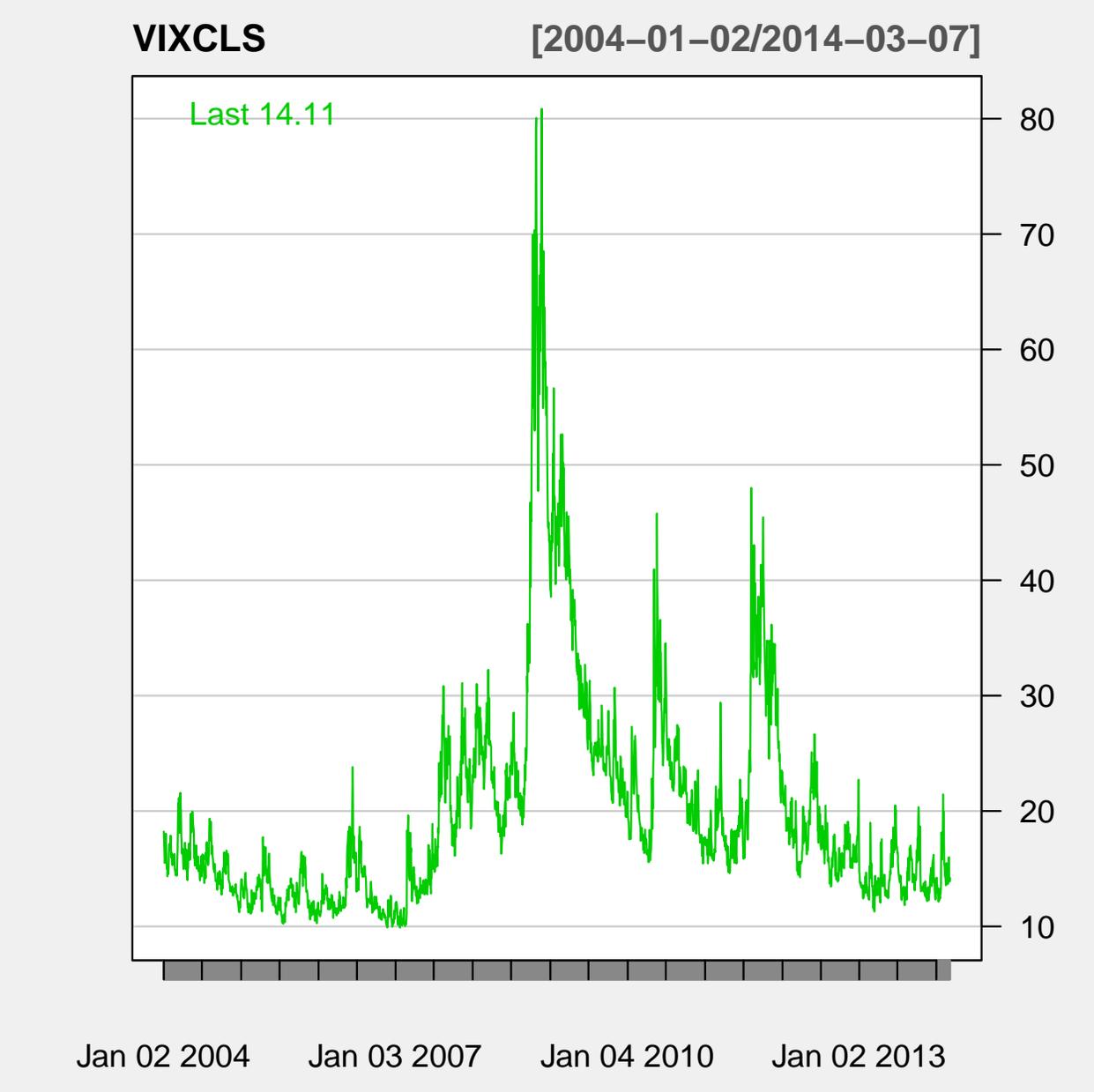


Figure 4: CBOE Vix index: January 2, 2004 to March 7, 2014.

EPS of Coca Cola: 1983–2009

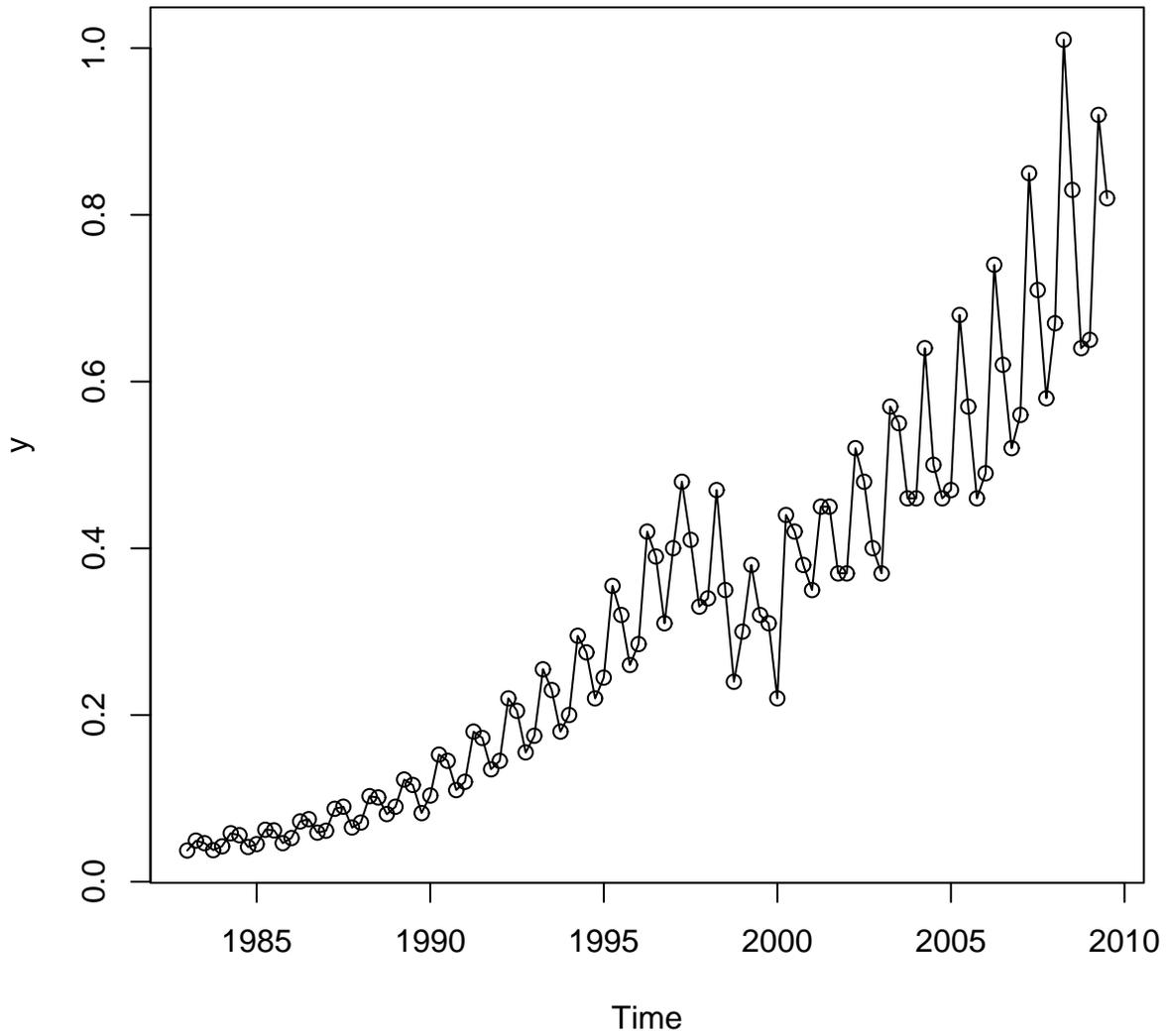


Figure 5: Quarterly earnings per share of Coca-Cola Company

Dollars per Euro

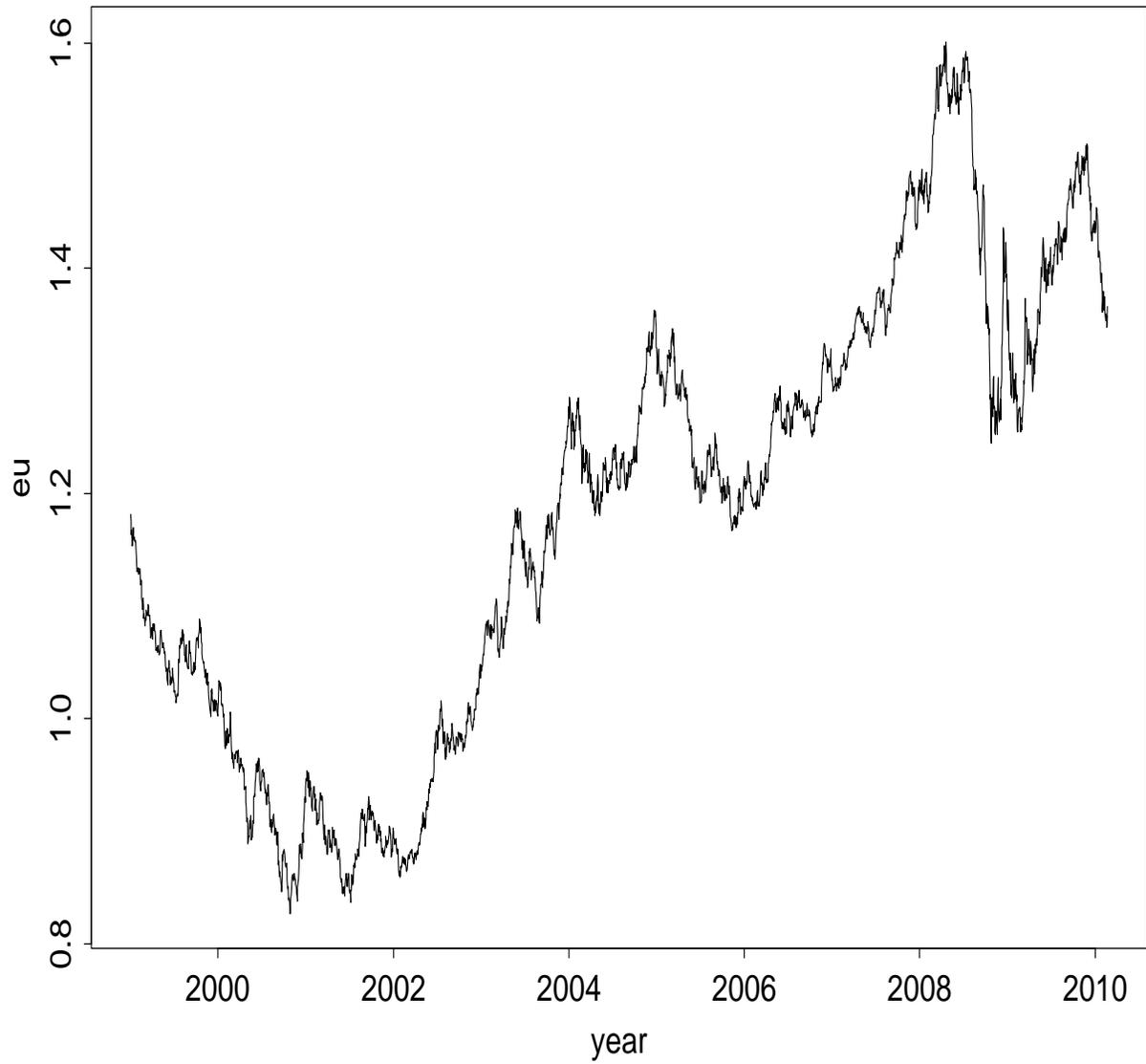


Figure 6: Daily Exchange Rate: Dollars per Euro

In-rtn: US-EU

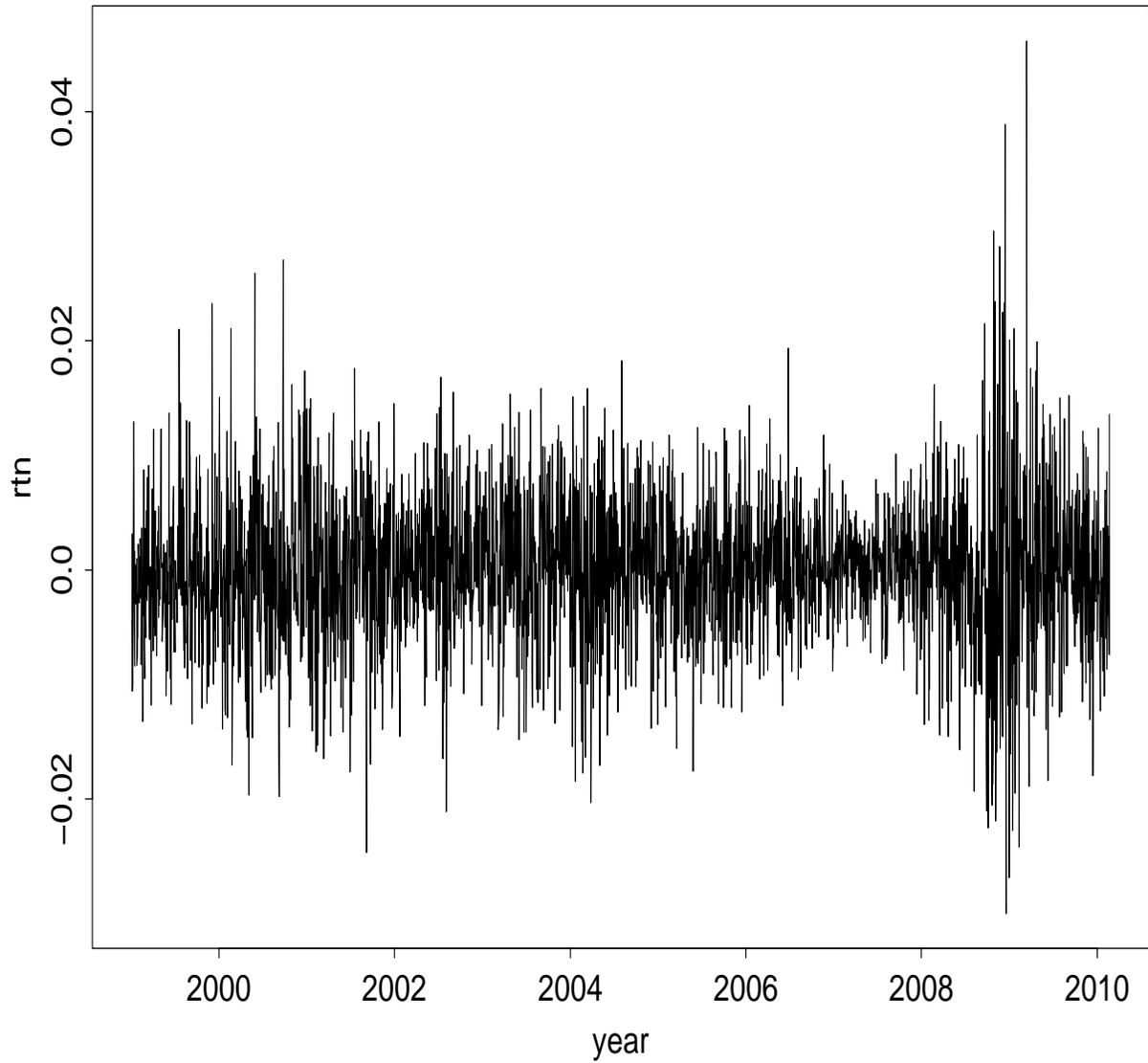


Figure 7: Daily log returns of FX (Dollar vs Euro)

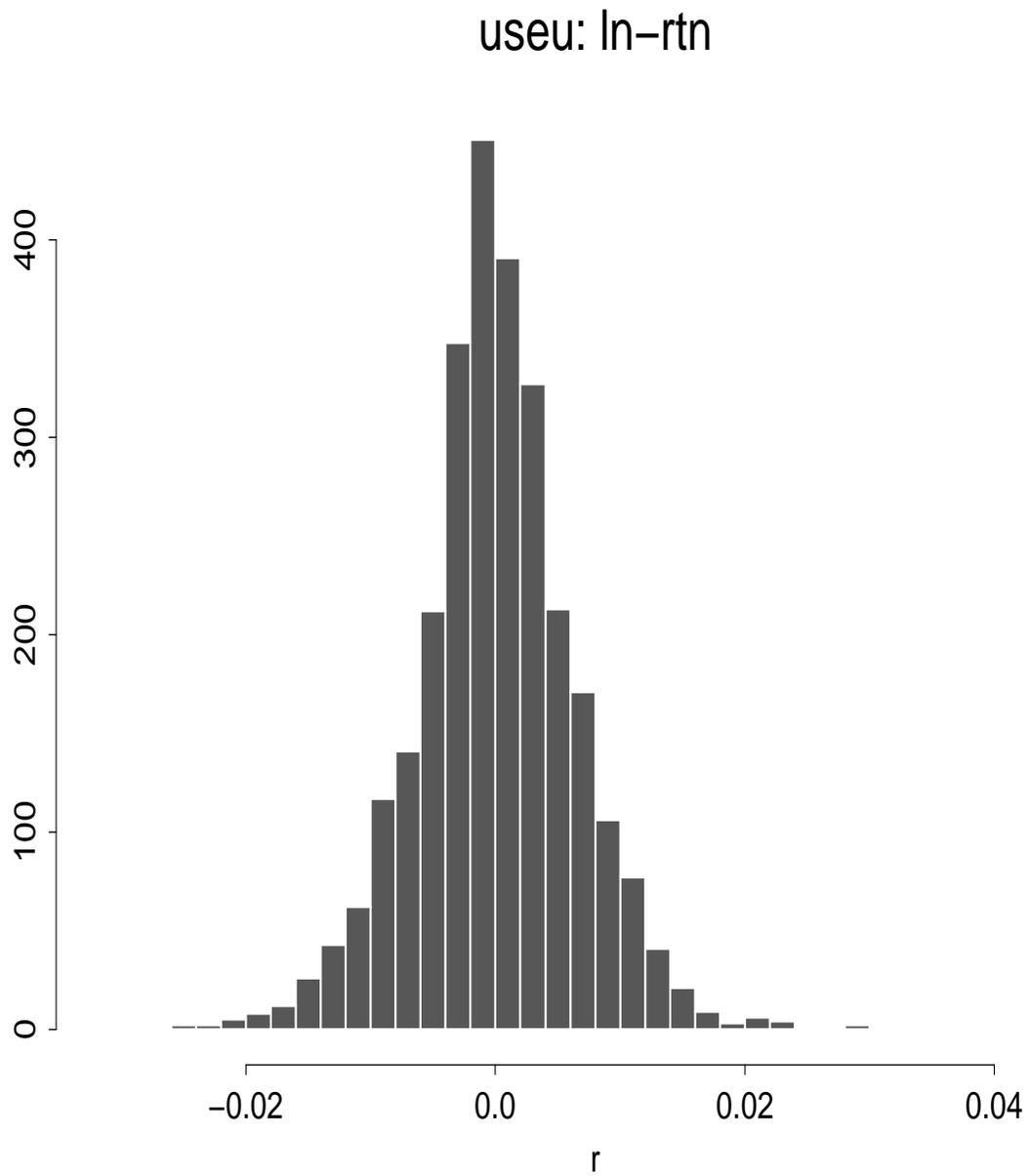


Figure 8: Histogram of daily log returns of FX (Dollar vs Euro)

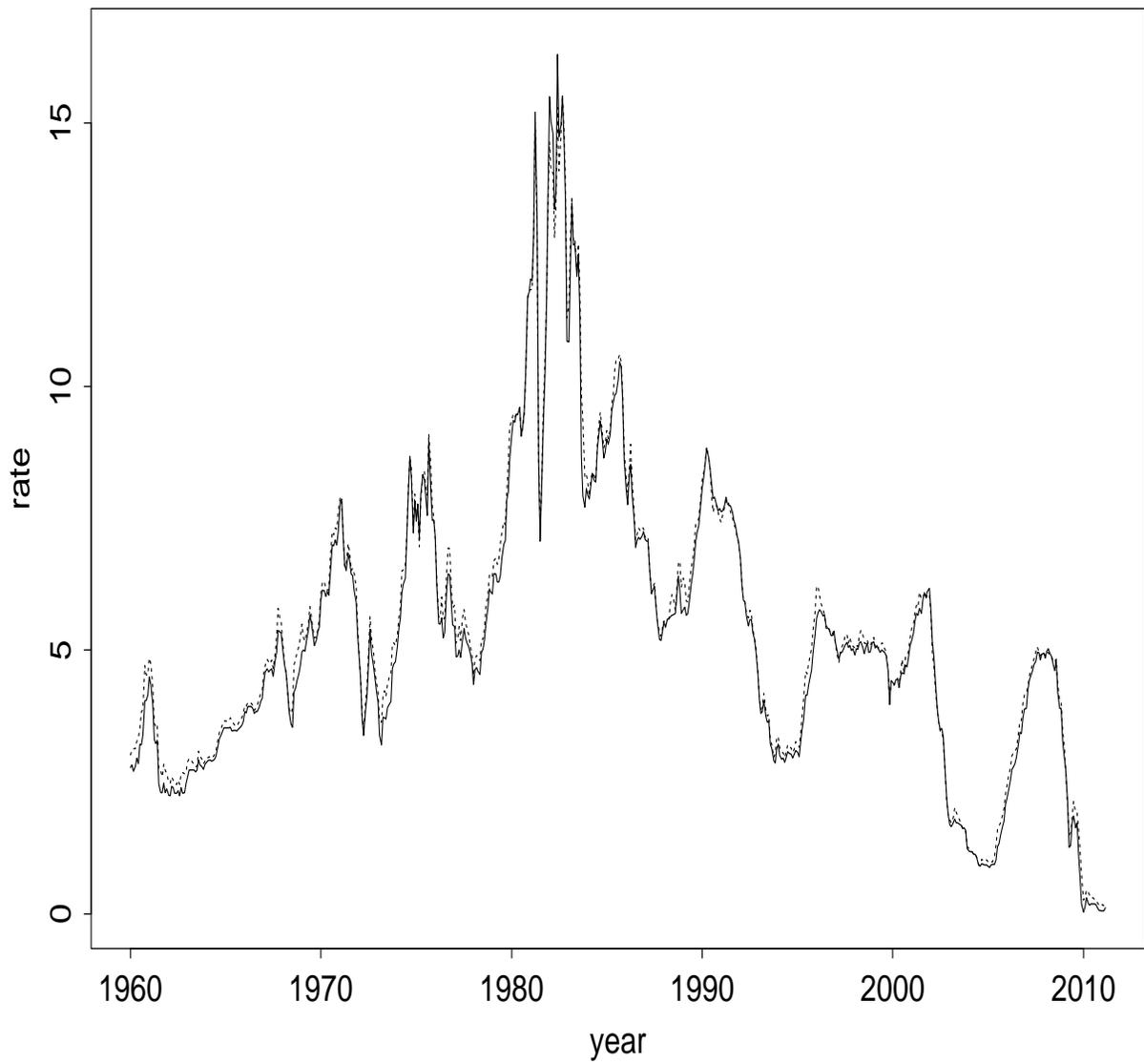


Figure 9: Monthly US interest rates: 3m & 6m TB

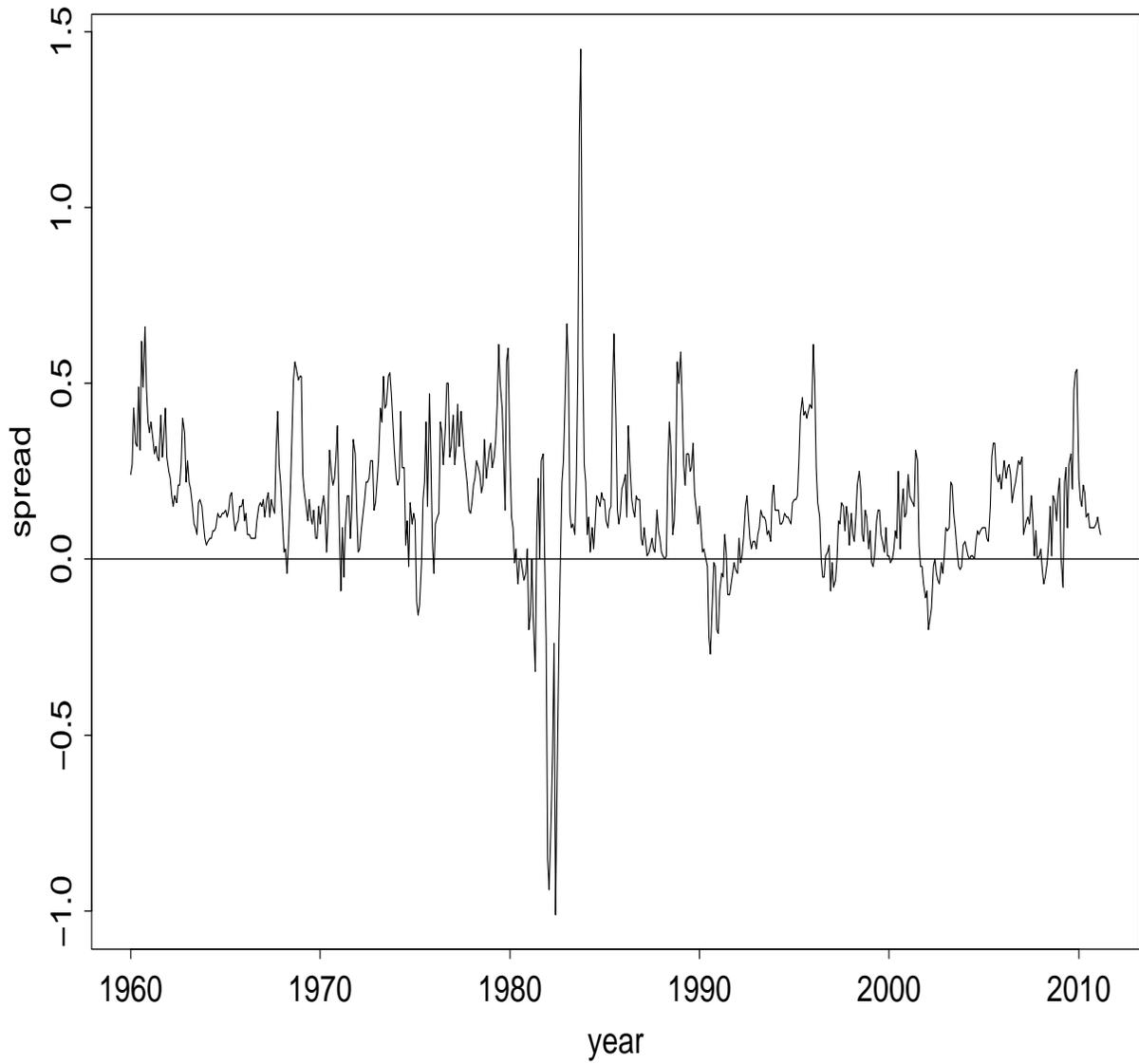


Figure 10: Spread of monthly US interest rates: 3m & 6m TB

Norwegian Fire Insurance Data: 1972–1992

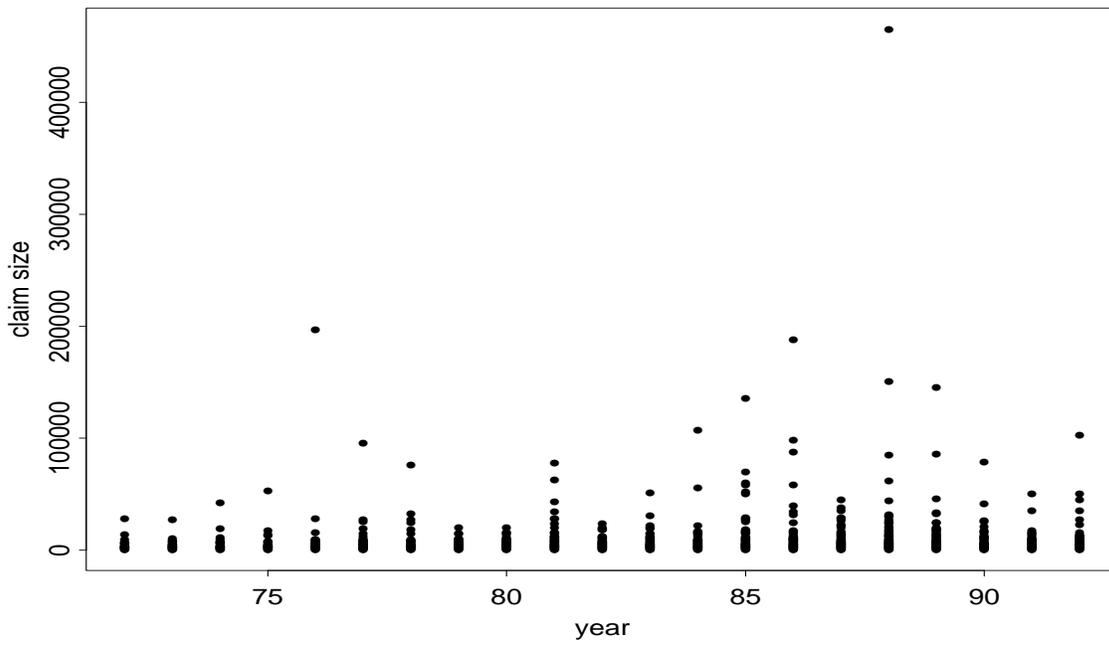


Figure 11: Claim sizes of the Norwegian fire insurance from 1972 to 1992, measured in 1000 Krone and exceeded 500.

CAT trade data on January 04, 2010.

```
date hour minute second price size
20100104 9 30 0 57.65 3910
20100104 9 30 0 57.7 400
20100104 9 30 0 57.68 100
20100104 9 30 0 57.69 300
20100104 9 30 1 57.65 462
20100104 9 30 1 57.65 100
20100104 9 30 1 57.65 100
20100104 9 30 1 57.65 100
20100104 9 30 1 57.7 100
20100104 9 30 1 57.7 100
20100104 9 30 1 57.72 500
20100104 9 30 1 57.72 100
20100104 9 30 2 57.73 100
20100104 9 30 3 57.73 300
20100104 9 30 3 57.72 100
20100104 9 30 4 57.72 300
20100104 9 30 5 57.57 100
20100104 9 30 5 57.57 500
20100104 9 30 5 57.56 300
.....
20100104 9 30 35 57.77 100
20100104 9 30 36 57.77 100
20100104 9 30 42 57.54 83600
20100104 9 30 42 57.57 100
.....
20100104 9 30 42 57.55 100
20100104 9 30 42 57.55 2400
20100104 9 30 42 57.56 100
20100104 9 30 42 57.55 100
20100104 9 30 42 57.55 100
20100104 9 30 42 57.55 100
20100104 9 30 42 57.54 170
20100104 9 30 42 57.54 200
```

Outline of the course

- Returns & their characteristics: empirical analysis (summary statistics)
- Simple linear time series models & their applications
- Univariate volatility models & their implications

- Nonlinearity in level and volatility
- Neural network & non-parametric methods
- High-frequency financial data and market micro-structure
- Continuous-time models and derivative pricing
- Value at Risk, extreme value theory and expected shortfall (also known as conditional VaR)
- Analysis of multiple asset returns: factor models, dynamic and cross dependence, cross-section regression

Asset Returns

Let P_t be the price of an asset at time t , and assume no dividend.

One-period simple return: Gross return

$$1 + R_t = \frac{P_t}{P_{t-1}} \quad \text{or} \quad P_t = P_{t-1}(1 + R_t)$$

Simple return:

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}.$$

Multiperiod simple return: Gross return

$$\begin{aligned} 1 + R_t(k) &= \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \dots \times \frac{P_{t-k+1}}{P_{t-k}} \\ &= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}). \end{aligned}$$

The k -period simple net return is $R_t(k) = \frac{P_t}{P_{t-k}} - 1$.

Example: Table below gives six daily (adjusted) closing prices of Apple stock in December 2015. The 1-day gross return of holding the

stock from 12/23 to 12/24 $1 + R_t = 107.45/108.02 \approx 0.9947$ so that the daily simple return is -0.53% , which is $(107.45 - 108.02)/108.02$.

Date	12/23	12/24	12/28	12/29	12/30	12/31
Price(\$)	108.02	107.45	106.24	108.15	106.74	104.69

Time interval is important! Default is one year.

Annualized (average) return:

$$\text{Annualized}[R_t(k)] = \left[\prod_{j=0}^{k-1} (1 + R_{t-j}) \right]^{1/k} - 1.$$

An approximation:

$$\text{Annualized}[R_t(k)] \approx \frac{1}{k} \sum_{j=0}^{k-1} R_{t-j}.$$

Continuously compounding: Illustration of the power of compounding (int. rate 10% per annum)

Type	#(payment)	Int.	Net
Annual	1	0.1	\$1.10000
Semi-Annual	2	0.05	\$1.10250
Quarterly	4	0.025	\$1.10381
Monthly	12	0.0083	\$1.10471
Weekly	52	$\frac{0.1}{52}$	\$1.10506
Daily	365	$\frac{0.1}{365}$	\$1.10516
Continuously	∞		\$1.10517

$$A = C \exp[r \times n]$$

where r is the interest rate per annum, C is the initial capital, n is the number of years, and \exp is the exponential function.

Present value:

$$C = A \exp[-r \times n]$$

Continuously compounded (or log) return

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} = p_t - p_{t-1},$$

where $p_t = \ln(P_t)$.

Multiperiod log return:

$$\begin{aligned} r_t(k) &= \ln[1 + R_t(k)] \\ &= \ln[(1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1})] \\ &= \ln(1 + R_t) + \ln(1 + R_{t-1}) + \cdots + \ln(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + \cdots + r_{t-k+1}. \end{aligned}$$

Example. Consider again the Apple stock price.

1. What is the log return from 12/23 to 12/24:

$$\text{A: } r_t = \ln(107.45) - \ln(108.02) = -0.529\%.$$

2. What is the log return from day 12/23 to 12/31?

$$\text{A: } r_t(6) = \ln(104.69) - \ln(108.02) = -3.13\%.$$

Portfolio return: N assets

$$R_{p,t} = \sum_{i=1}^N w_i R_{it}$$

Example: An investor holds stocks of IBM, Microsoft and Citi-Group. Assume that her capital allocation is 30%, 30% and 40%. Use the monthly simple returns in Table 1.2 of the text. What is the mean simple return of her stock portfolio?

Answer: $E(R_t) = 0.3 \times 1.35 + 0.3 \times 2.62 + 0.4 \times 1.17 = 1.66$.

Dividend payment:

$$R_t = \frac{P_t + D_t}{P_{t-1}} - 1, \quad r_t = \ln(P_t + D_t) - \ln(P_{t-1}).$$

Excess return: (adjusting for risk)

$$Z_t = R_t - R_{0t}, \quad z_t = r_t - r_{0t}$$

where r_{0t} denotes the log return of a reference asset (e.g. risk-free interest rate).

Relationship:

$$r_t = \ln(1 + R_t), \quad R_t = e^{r_t} - 1.$$

If the returns are in **percentage**, then

$$r_t = 100 \times \ln\left(1 + \frac{R_t}{100}\right), \quad R_t = [\exp(r_t/100) - 1] \times 100.$$

Temporal aggregation of the returns produces

$$\begin{aligned} 1 + R_t(k) &= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}), \\ r_t(k) &= r_t + r_{t-1} + \cdots + r_{t-k+1}. \end{aligned}$$

These two relations are important in practice, e.g. obtain annual returns from monthly returns.

Example. If the monthly log returns of an asset are 4.46%, -7.34% and 10.77%, then what is the corresponding quarterly log return?

Answer: $4.46 - 7.34 + 10.77 = 7.89\%$.

Example: If the monthly simple returns of an asset are 4.46%, -7.34% and 10.77%, then what is the corresponding quarterly simple return?

Answer: $R = (1 + 0.0446)(1 - 0.0734)(1 + 0.1077) - 1 = 1.0721 - 1 = 0.0721 = 7.21\%$

Distributional properties of returns

Key: What is the distribution of

$\{r_{it}; i = 1, \dots, N; t = 1, \dots, T\}$?

Some theoretical properties:

Moments of a random variable X with density $f(x)$: ℓ -th moment

$$m'_\ell = E(X^\ell) = \int_{-\infty}^{\infty} x^\ell f(x) dx$$

First moment: mean or expectation of X .

ℓ -th central moment

$$m_\ell = E[(X - \mu_x)^\ell] = \int_{-\infty}^{\infty} (x - \mu_x)^\ell f(x) dx,$$

2nd central moment: **Variance** of X .

standard deviation: square-root of variance

Skewness (symmetry) and kurtosis (fat-tails)

$$S(x) = E \left[\frac{(X - \mu_x)^3}{\sigma_x^3} \right], \quad K(x) = E \left[\frac{(X - \mu_x)^4}{\sigma_x^4} \right].$$

$K(x) - 3$: **Excess kurtosis.**

Q1: Why study the mean and variance of returns?

They are concerned with long-term return and risk, respectively.

Q2: Why is symmetry important?

Symmetry has important implications in holding short or long financial positions and in risk management.

Q3: Why is kurtosis important?

Related to volatility forecasting, efficiency in estimation and tests
High kurtosis implies heavy (or long) tails in distribution.

Estimation:

Data: $\{x_1, \dots, x_T\}$

- sample mean:

$$\hat{\mu}_x = \frac{1}{T} \sum_{t=1}^T x_t,$$

- sample variance:

$$\hat{\sigma}_x^2 = \frac{1}{T-1} \sum_{t=1}^T (x_t - \hat{\mu}_x)^2,$$

- sample skewness:

$$\hat{S}(x) = \frac{1}{(T-1)\hat{\sigma}_x^3} \sum_{t=1}^T (x_t - \hat{\mu}_x)^3,$$

- sample kurtosis:

$$\hat{K}(x) = \frac{1}{(T-1)\hat{\sigma}_x^4} \sum_{t=1}^T (x_t - \hat{\mu}_x)^4.$$

Under normality assumption,

$$\hat{S}(x) \sim N\left(0, \frac{6}{T}\right), \quad \hat{K}(x) - 3 \sim N\left(0, \frac{24}{T}\right).$$

Some simple tests for normality (for large T).

1. Test for symmetry:

$$S^* = \frac{\hat{S}(x)}{\sqrt{6/T}} \sim N(0, 1)$$

if normality holds.

Decision rule: Reject H_0 of a symmetric distribution if $|S^*| > Z_{\alpha/2}$ or p-value is less than α .

2. Test for tail thickness:

$$K^* = \frac{\hat{K}(x) - 3}{\sqrt{24/T}} \sim N(0, 1)$$

if normality holds.

Decision rule: Reject H_o of normal tails if $|K^*| > Z_{\alpha/2}$ or p-value is less than α .

3. A joint test (Jarque-Bera test):

$$JB = (K^*)^2 + (S^*)^2 \sim \chi_2^2$$

if normality holds, where χ_2^2 denotes a chi-squared distribution with 2 degrees of freedom.

Decision rule: Reject H_o of normality if $JB > \chi_2^2(\alpha)$ or p-value is less than α .

Empirical properties of returns

Data sources: Use packages, e.g. [quantmod](#)

- Course web:
- CRSP: Center for Research in Security Prices (Wharton WRDS)
<https://wrds-web.wharton.upenn.edu/wrds/>
- Various web sites, e.g. Federal Reserve Bank at St. Louis
<https://research.stlouisfed.org/fred2/>
- Data sets of the textbook:
<http://faculty.chicagobooth.edu/ruey.tsay/teaching/fts3/>

Empirical dist of asset returns tends to be skewed to the left with heavy tails and has a higher peak than normal dist. See Table 1.2 of the text.

Demonstration of Data Analysis

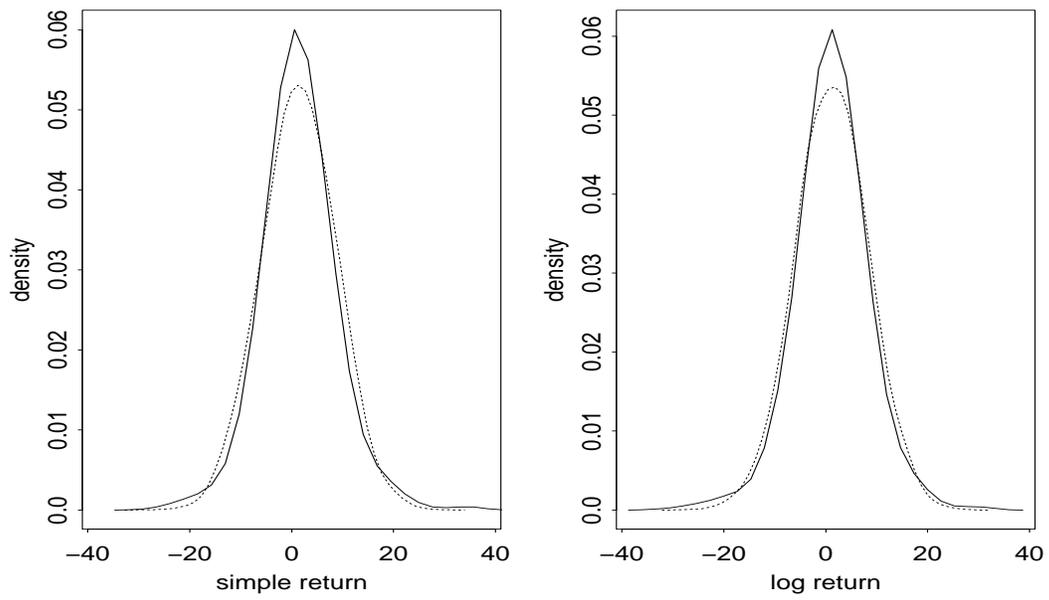


Figure 12: Comparison of empirical IBM return densities (solid) with Normal densities (dashed)

R demonstration: Use monthly IBM stock returns from 1967 to 2008.

- **** Task: (a) Set the working directory
(b) Load the library ‘fBasics’.
(c) Compute summary (or descriptive) statistics
(d) Perform test for mean return being zero.
(e) Perform normality test using the Jaque-Bera method.
(f) Perform skewness and kurtosis tests.

```
> setwd("...")                <== set working directory
> library(fBasics)           <== Load the library ‘fBasics’.
```



```
> da=read.table("m-ibm-6815.txt",header=T)
> head(da)
  PERMNO      date    PRC ASKHI  BIDLO      RET    vwretd    ewretd    sprtrn
1  12490 19680131 594.50 623.0 588.75 -0.051834 -0.036330  0.023902 -0.043848
2  12490 19680229 580.00 599.5 571.00 -0.022204 -0.033624 -0.056118 -0.031223
3  12490 19680329 612.50 612.5 562.00  0.056034  0.005116 -0.011218  0.009400
4  12490 19680430 677.50 677.5 630.00  0.106122  0.094148  0.143031  0.081929
5  12490 19680531 357.00 696.0 329.50  0.055793  0.027041  0.091309  0.011169
6  12490 19680628 353.75 375.0 346.50 -0.009104  0.011527  0.016225  0.009120
```



```
> dim(da)
[1] 576  9
```

```
> ibm=da$RET % Simple IBM return
> lnIBM <- log(ibm+1) % compute log return
> ts.plot(ibm,main="Monthly IBM simple returns: 1968-2015") % Time plot
> mean(ibm)
[1] 0.008255663
> var(ibm)
[1] 0.004909968
> skewness(ibm)
[1] 0.2687105
attr(,"method")
[1] "moment"
> kurtosis(ibm)
[1] 2.058484
attr(,"method")
[1] "excess"
> basicStats(ibm)
```

	ibm
nobs	576.000000
NAs	0.000000
Minimum	-0.261905
Maximum	0.353799
1. Quartile	-0.034392

```

3. Quartile    0.048252
Mean           0.008256
Median         0.005600
Sum            4.755262
SE Mean        0.002920
LCL Mean       0.002521
UCL Mean       0.013990
Variance       0.004910
Stdev          0.070071
Skewness       0.268710
Kurtosis       2.058484
> basicStats(lnIBM) % log return
              lnIBM
nobs          576.000000
NAs            0.000000
Minimum        -0.303683
Maximum         0.302915
1. Quartile    -0.034997
3. Quartile     0.047124
Mean            0.005813
Median          0.005585
Sum             3.348008
SE Mean         0.002898
LCL Mean        0.000120
UCL Mean        0.011505
Variance        0.004839
Stdev           0.069560
Skewness        -0.137286
Kurtosis        1.910438
> t.test(lnIBM) %% Test mean=0 vs mean .not. zero

```

One Sample t-test

```

data: lnIBM
t = 2.0055, df = 575, p-value = 0.04538
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.0001199015 0.0115051252
sample estimates:
 mean of x
0.005812513

```

```

> normalTest(lnIBM,method='jb')
Title: Jarque - Bera Normalality Test

```

Test Results:

```

STATISTIC:
  X-squared: 90.988
P VALUE:
  Asymptotic p Value: < 2.2e-16

> s3=skewness(lnIBM); T <- length(lnIBM)
> tst <- s3/sqrt(6/T) % test skewness
> tst
[1] -1.345125
> pv <- 2*pnorm(tst)
> pv
[1] 0.1785849
> k4 <- kurtosis(lnIBM)
> tst <- k4/sqrt(24/T) % test excess kurtosis
> tst
[1] 9.359197
>q() % quit R.

```

Normal and lognormal dists

Y is lognormal if $X = \ln(Y)$ is normal.

If $X \sim N(\mu, \sigma^2)$, then $Y = \exp(X)$ is lognormal with

Mean and variance:

$$E(Y) = \exp\left(\mu + \frac{\sigma^2}{2}\right), \quad V(Y) = \exp(2\mu + \sigma^2)[\exp(\sigma^2) - 1].$$

Conversely, if Y is lognormal with mean μ_y and variance σ_y^2 , then $X = \ln(Y)$ is normal with mean and variance

$$E(X) = \ln \left[\frac{\mu_y}{\sqrt{1 + \frac{\sigma_y^2}{\mu_y^2}}} \right], \quad V(X) = \ln \left[1 + \frac{\sigma_y^2}{\mu_y^2} \right].$$

Application: If the log return of an asset is normally distributed with mean 0.0119 and standard deviation 0.0663, then what is the mean and standard deviation of its simple return?

Answer: Solve this problem in two steps.

Step 1: Based on the prior results, the mean and variance of $Y_t = \exp(r_t)$ are

$$E(Y) = \exp \left[0.0119 + \frac{0.0663^2}{2} \right] = 1.014$$

$$V(Y) = \exp(2 \times 0.0119 + 0.0663^2) [\exp(0.0663^2) - 1] = 0.0045$$

Step 2: Simple return is $R_t = \exp(r_t) - 1 = Y_t - 1$. Therefore,

$$E(R) = E(Y) - 1 = 0.014$$

$$V(R) = V(Y) = 0.0045, \quad \text{standard dev} = \sqrt{V(R)} = 0.067$$

Remark: See the monthly IBM stock returns in Table 1.2.

Processes considered

- return series (e.g., ch. 1, 2, 5)
- volatility processes (e.g., ch. 3, 4, 10, 12)
- continuous-time processes (ch. 6)
- extreme events (ch. 7)
- multivariate series (ch. 8, 9, 10)

Likelihood function (for self study)

Finally, it pays to study the likelihood function of returns $\{r_1, \dots, r_T\}$ discussed in Chapter 1.

Basic concept:

Joint dist = Conditional dist \times Marginal dist, i.e.

$$f(x, y) = f(x|y)f(y)$$

For two consecutive returns r_1 and r_2 , we have

$$f(r_2, r_1) = f(r_2|r_1)f(r_1).$$

For three returns r_1 , r_2 and r_3 , by repeated application,

$$\begin{aligned} f(r_3, r_2, r_1) &= f(r_3|r_2, r_1)f(r_2, r_1) \\ &= f(r_3|r_2, r_1)f(r_2|r_1)f(r_1). \end{aligned}$$

In general, we have

$$\begin{aligned} &f(r_T, r_{T-1}, \dots, r_2, r_1) \\ &= f(r_T|r_{T-1}, \dots, r_1)f(r_{T-1}, \dots, r_1) \\ &= f(r_T|r_{T-1}, \dots, r_1)f(r_{T-1}|r_{T-2}, \dots, r_1)f(r_{T-2}, \dots, r_1) \\ &= \vdots \\ &= \left[\prod_{t=2}^T f(r_t|r_{t-1}, \dots, r_1) \right] f(r_1), \end{aligned}$$

where $\prod_{t=2}^T$ denotes product.

If $r_t|r_{t-1}, \dots, r_1$ is normal with mean μ_t and variance σ_t^2 , then likelihood function becomes

$$f(r_T, r_{T-1}, \dots, r_1) = \prod_{t=2}^T \frac{1}{\sqrt{2\pi}\sigma_t} \exp \left[\frac{-(r_t - \mu_t)^2}{2\sigma_t^2} \right] f(r_1).$$

For simplicity, if $f(r_1)$ is ignored, then the likelihood function becomes

$$f(r_T, r_{T-1}, \dots, r_1) = \prod_{t=2}^T \frac{1}{\sqrt{2\pi}\sigma_t} \exp \left[\frac{-(r_t - \mu_t)^2}{2\sigma_t^2} \right].$$

This is the *conditional* likelihood function of the returns under normality.

Other dists, e.g. Student- t , can be used to handle heavy tails.

Model specification

- μ_t : discussed in Chapter 2
- σ_t^2 : Chapters 3 and 4.

Quantifying dependence: Consider two variables X and Y .

- **Pearson's correlation coefficient:**

$$\rho = \frac{\text{Cov}(X, Y)}{\text{std}(X)\text{std}(Y)}.$$

- **Kendall's tau:** Let (\tilde{X}, \tilde{Y}) be a random copy of (X, Y) .

$$\begin{aligned} \rho_\tau &= P[(X - \tilde{X})(Y - \tilde{Y}) > 0] - P[(X - \tilde{X})(Y - \tilde{Y}) < 0] \\ &= E[\text{sign}[(X - \tilde{X})(Y - \tilde{Y})]]. \end{aligned}$$

This measure quantifies the probability of *concordant* over *discordant*. Here concordant means $(X - \tilde{X})(Y - \tilde{Y}) > 0$. For spherical distributions, e.g., normal, $\rho_\tau = \frac{2}{\pi} \sin^{-1}(\rho)$.

- **Spearman's rho:** rank correlation. Let $F_x(x)$ and $F_y(y)$ be the cumulative distribution function of X and Y .

$$\rho_s = \rho(F_x(X), F_y(Y)).$$

That is, the correlation coefficient of probability-transformed variables. It is just the correlation coefficient of the **ranks** of the data.

Q: Why do we consider different measures of dependence?

- Correlation coefficient encounters problems when the distributions are not normal (spherical, in general). This is particularly relevant in risk management.

- Correlation coefficient focuses on linear dependence and is not robust to outliers.
- The actual range of the correlation coefficient can be much smaller than $[-1, 1]$.

R Demonstration

```
> head(da)
  PERMNO      date    PRC ASKHI  BIDLO      RET    vwretd    ewretd    sprtrn
1  12490 19680131 594.50 623.0 588.75 -0.051834 -0.036330  0.023902 -0.043848
2  12490 19680229 580.00 599.5 571.00 -0.022204 -0.033624 -0.056118 -0.031223
3  12490 19680329 612.50 612.5 562.00  0.056034  0.005116 -0.011218  0.009400
4  12490 19680430 677.50 677.5 630.00  0.106122  0.094148  0.143031  0.081929
5  12490 19680531 357.00 696.0 329.50  0.055793  0.027041  0.091309  0.011169
6  12490 19680628 353.75 375.0 346.50 -0.009104  0.011527  0.016225  0.009120
> ibm <- da$RET
> sp <- da$sprtrn
> plot(sp, ibm)
> cor(sp, ibm)
[1] 0.5785249
> cor(sp, ibm, method="kendall")
[1] 0.4172056
> cor(sp, ibm, method="spearman")
[1] 0.58267
> cor(rank(ibm), rank(sp))
[1] 0.58267

> z=rnorm(1000) %% Generate 1000 random variates from N(0,1)
> x=exp(z)
> y=exp(20*z)
> cor(x, y)
[1] 0.3187030
> cor(x, y, method='kendall')
[1] 1
> cor(x, y, method='spearman')
[1] 1
```

Takeaway

1. Understand the summary statistics of asset returns
2. Understand various definitions of returns & their relationships

3. Learn basic characteristics of FTS

4. Learn the basic **R** functions. (See Rcommands-lec1.txt on the course web.)

R commands used to produce plots in Lecture 1.

```
> x=read.table("d-aapl0413.txt",header=T)  <== Load Apple stock returns
> dim(x)          <== check the size of the data file
[1] 2517 3
> x[1,]          <== show the first row of the data
  Permno      date      rtn
1  14593 20040102 -0.004212
> y=ts(x[,3],frequency=252,start=c(2004,1))  <== Create a time-series object in R.
> plot(y,type='l',xlab='year',ylab='rtn')
> title(main='Daily returns of Apple stock: 2004 to 2013')
```



```
> par(mfcol=c(2,1))  <== To put two plots on a single page
> y=y*100            <== percentage returns
> hist(y,nclass=50)
> title(main='Percentage returns')
> d1=density(y)
> plot(d1$x,d1$y,xlab='returns',ylab='den',type='l')
```



```
> x=read.table("m-tb3ms.txt",header=T)  <== Load 3m-TB rates
> dim(x)
[1] 914  4
```



```
> y=read.table("m-tb6ms.txt",header=T)  <== Load 6m-TB rates
> dim(y)
[1] 615  4
> 914-615
[1] 299
> x[300,]  <== Check date of the 3m-TB
  year mon day value
300 1958 12  1  2.77
> y[1,]    <== Check date of the 1st observation of 6m-TB
  year mon day value
1 1958 12  1  3.01
```



```
> int=cbind(x[300:914,4],y[,4])  <== Line up the two TB rates
> tdx=(c(1:615)+11)/12+1959
> par(mfcol=c(1,1))
> max(int)
```

```

[1] 16.3
> plot(tdx,int[,1],xlab='year',ylab='rate',type='l',ylim=c(0,16.5))
> lines(tdx,int[,2],lty=2)    <== Plot the 6m-TB rate on the same frame.

> plot(tdx,int[,2]-int[,1],xlab='year',ylab='spread',type='l')
> abline(h=c(0))    <== Draw a horizontal line to 'zero'.

> x=read.table("q-ko-earnings8309.txt",header=T)    <== Load KO data
> dim(x)
[1] 107    3
> x[1,]
      pends  anntime  value
1 19830331 19830426 0.0375
> tdx=c(1:107)/12+1983
> plot(tdx,x[,3],xlab='year',ylab='earnings',type='l')
> title(main='EPS of Coca Cola: 1983-2009')
> points(tdx,x[,3])
>
> y=read.table("d-exuseu.txt",header=T)    <== Load USEU exchange rates
> dim(y)
[1] 3567    4
> y[1,]
      year mon day  value
1 1999    1    4 1.1812
> tdx=c(1:3567)/252+1999
> plot(tdx,y[,4],xlab='year',ylab='eu',type='l')
> title(main='Dollars per Euro')

> r=diff(log(y[,4]))    <== Compute log returns
> plot(tdx[2:3567],r,xlab='year',ylab='rtn',type='l')
> title(main='ln-rtn: US-EU')

> hist(r,nclass=50)
> title(main='useu: ln-rtn')

```

Linear Time Series (TS) Models

Financial TS: collection of a financial measurement over time

Example: log return r_t

Data: $\{r_1, r_2, \dots, r_T\}$ (T data points)

Purpose: What is the information contained in $\{r_t\}$?

Basic concepts

- Stationarity:
 - Strict: distributions are time-invariant
 - Weak: first 2 moments are time-invariant

What does weak stationarity mean in practice?

Past: time plot of $\{r_t\}$ varies around a fixed level within a finite range!

Future: the first 2 moments of future r_t are the same as those of the data so that meaningful inferences can be made.

- Mean (or expectation) of returns:

$$\mu = E(r_t)$$

- Variance (variability) of returns:

$$\text{Var}(r_t) = E[(r_t - \mu)^2]$$

- Sample mean and sample variance are used to estimate the mean and variance of returns.

$$\bar{r} = \frac{1}{T} \sum_{t=1}^T r_t \quad \& \quad \text{Var}(r_t) = \frac{1}{T-1} \sum_{t=1}^T (r_t - \bar{r})^2$$

- Test $H_o : \mu = 0$ vs $H_a : \mu \neq 0$. Compute

$$t = \frac{\bar{r}}{\text{std}(\bar{r})} = \frac{\bar{r}}{\sqrt{\text{Var}(r_t)/T}}$$

Compare t ratio with $N(0, 1)$ dist.

Decision rule: Reject H_o of zero mean if $|t| > Z_{\alpha/2}$ or p-value is less than α .

- Lag- k autocovariance:

$$\gamma_k = \text{Cov}(r_t, r_{t-k}) = E[(r_t - \mu)(r_{t-k} - \mu)].$$

- Serial (or auto-) correlations:

$$\rho_\ell = \frac{\text{cov}(r_t, r_{t-\ell})}{\text{var}(r_t)}$$

Note: $\rho_0 = 1$ and $\rho_k = \rho_{-k}$ for $k \neq 0$. Why?

Existence of serial correlations implies that the return is predictable, indicating market inefficiency.

- Sample autocorrelation function (ACF)

$$\hat{\rho}_\ell = \frac{\sum_{t=1}^{T-\ell} (r_t - \bar{r})(r_{t+\ell} - \bar{r})}{\sum_{t=1}^T (r_t - \bar{r})^2},$$

where \bar{r} is the sample mean & T is the sample size.

- Test zero serial correlations (market efficiency)

– Individual test: for example,

$$H_o : \rho_1 = 0 \text{ vs } H_a : \rho_1 \neq 0$$

$$t = \frac{\hat{\rho}_1}{\sqrt{1/T}} = \sqrt{T} \hat{\rho}_1$$

Asym. $N(0, 1)$.

Decision rule: Reject H_o if $|t| > Z_{\alpha/2}$ or p-value less than α .

– Joint test (Ljung-Box statistics):

$H_o : \rho_1 = \dots = \rho_m = 0$ vs $H_a : \rho_i \neq 0$

$$Q(m) = T(T + 2) \sum_{\ell=1}^m \frac{\hat{\rho}_\ell^2}{T - \ell}$$

Asym. chi-squared dist with m degrees of freedom.

Decision rule: Reject H_o if $Q(m) > \chi_m^2(\alpha)$ or p-value is less than α .

- Sources of serial correlations in financial TS
 - Nonsynchronous trading (ch. 5)
 - Bid-ask bounce (ch. 5)
 - Risk premium, etc. (ch. 3)

Thus, significant sample ACF does not necessarily imply market inefficiency.

Example: Monthly returns of IBM stock from 1926 to 1997.

- R_t : $Q(5) = 5.4(0.37)$ and $Q(10) = 14.1(0.17)$
- r_t : $Q(5) = 5.8(0.33)$ and $Q(10) = 13.7(0.19)$

Remark: What is p-value? How to use it?

Implication: Monthly IBM stock returns do not have significant serial correlations.

Example: Monthly returns of CRSP value-weighted index from 1926 to 1997.

- R_t : $Q(5) = 27.8$ and $Q(10) = 36.0$
- r_t : $Q(5) = 26.9$ and $Q(10) = 32.7$

All highly significant. Implication: there exist significant serial correlations in the value-weighted index returns. (Nonsynchronous trading might explain the existence of the serial correlations, among other reasons.) Similar result is also found in equal-weighted index returns.

R demonstration: IBM monthly simple returns from 1968 to 2015

```
> da=read.table("m-ibm-6815.txt",header=T)
> ibm=da$RET
> acf(ibm) %% Plot not shown
> m1 <- acf(ibm)
> names(m1)
[1] "acf"      "type"     "n.used"  "lag"     "series"  "snames"
> m1$acf
      [,1]
[1,] 1.0000000000 % lag 0
[2,] -0.0068713539 % lag 1
[3,] -0.0002212888
....
[28,] 0.0159729906

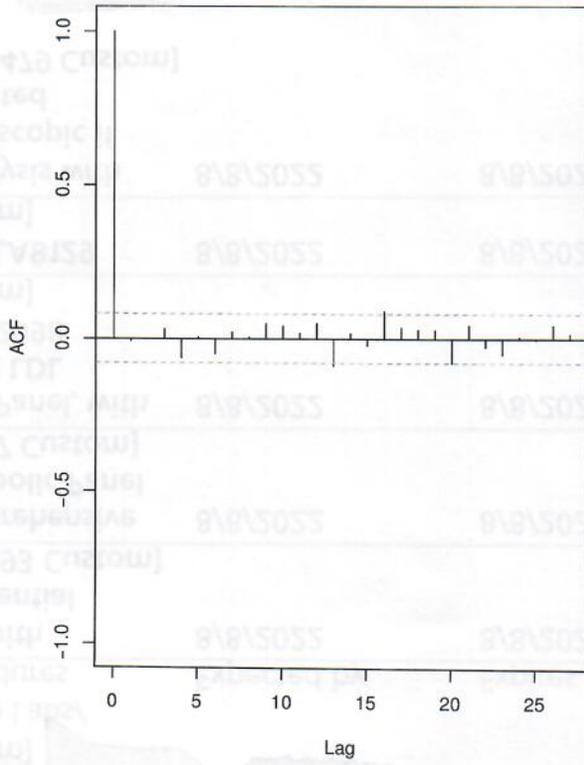
> m2 <- pacf(ibm) % Partial ACF
> names(m2)
[1] "acf"      "type"     "n.used"  "lag"     "series"  "snames"
> m1$acf
      [,1]
[1,] 1.0000000000
[2,] -0.0068713539
[3,] -0.0002212888
....
[28,] 0.0159729906

> Box.test(ibm,lag=10) % Box-Pierce Q(m) test
      Box-Pierce test
data:  ibm
X-squared = 7.1714, df = 10, p-value = 0.7092

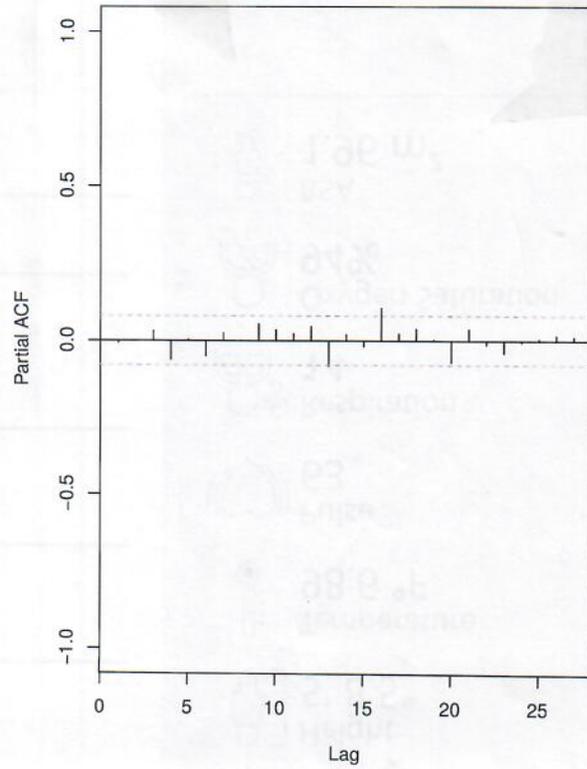
> Box.test(ibm,lag=10,type='Ljung') % Ljung-Box Q(m) test
      Box-Ljung test
data:  ibm
```

X-squared = 7.2759, df = 10, p-value = 0.6992

Series ibm



Series ibm



35 $\frac{1}{2}$

X-squared = 7.2759, df = 10, p-value = 0.6992

Back-shift (lag) operator

A useful notation in TS analysis.

- Definition: $Br_t = r_{t-1}$ or $Lr_t = r_{t-1}$
- $B^2r_t = B(Br_t) = Br_{t-1} = r_{t-2}$.

B (or L) means time shift! Br_t is the value of the series at time $t - 1$.

Suppose that the daily log returns are

Day	1	2	3	4
r_t	0.017	-0.005	-0.014	0.021

Answer the following questions:

- $r_2 =$
- $Br_3 =$
- $B^2r_5 =$

Question: What is B^2 ?

What are the important statistics in practice?

Conditional quantities, not unconditional

A proper perspective: at a time point t

- Available data: $\{r_1, r_2, \dots, r_{t-1}\} \equiv F_{t-1}$
- The return is decomposed into two parts as

$$\begin{aligned} r_t &= \text{predictable part} + \text{not predictable part} \\ &= \text{function of elements of } F_{t-1} + a_t \end{aligned}$$

In other words, given information F_{t-1}

$$\begin{aligned}r_t &= \mu_t + a_t \\ &= E(r_t|F_{t-1}) + \sigma_t\epsilon_t\end{aligned}$$

- μ_t : conditional mean of r_t
- a_t : shock or innovation at time t
- ϵ_t : an iid sequence with mean zero and variance 1
- σ_t : conditional standard deviation (commonly called volatility in finance)

Traditional TS modeling is concerned with μ_t :

Model for μ_t : **mean equation**

Volatility modeling concerns σ_t .

Model for σ_t^2 : **volatility equation**

Univariate TS analysis serves two purposes

- a model for μ_t
- understanding models for σ_t^2 : properties, forecasting, etc.

Linear time series: r_t is linear if

- the predictable part is a linear function of F_{t-1}
- $\{a_t\}$ are independent and have the same dist. (iid)

Mathematically, it means r_t can be written as

$$r_t = \mu + \sum_{i=0}^{\infty} \psi_i a_{t-i},$$

where μ is a constant, $\psi_0 = 1$ and $\{a_t\}$ is an iid sequence with mean zero and well-defined distribution.

In the economic literature, a_t is the *shock* (or *innovation*) at time t and $\{\psi_i\}$ are the *impulse* responses of r_t .

White noise: iid sequence (with finite variance), which is the building block of linear TS models.

White noise is not predictable, but has zero mean and finite variance.

Univariate linear time series models

1. autoregressive (AR) models
2. moving-average (MA) models
3. mixed ARMA models
4. seasonal models
5. regression models with time series errors
6. fractionally differenced models (long-memory)

Example Quarterly growth rate of U.S. real gross national product (GNP), seasonally adjusted, from the second quarter of 1947 to the first quarter of 1991.

An AR(3) model for the data is

$$r_t = 0.005 + 0.35r_{t-1} + 0.18r_{t-2} - 0.14r_{t-3} + a_t, \quad \hat{\sigma}_a = 0.01,$$

where $\{a_t\}$ denotes a white noise with variance σ_a^2 . Given r_n, r_{n-1} & r_{n-2} , we can predict r_{n+1} as

$$\hat{r}_{n+1} = 0.005 + 0.35r_n + 0.18r_{n-1} - 0.14r_{n-2}.$$

Other implications of the model?

In this course, we use *statistical methods* to find models that fit the data well for making inference, e.g. prediction. On the other hand, there exists economic theory that leads to time-series models for economic variables. For instance, consider the *real business-cycle theory* in macroeconomics. Under some simplifying assumptions, one can show that $\ln(Y_t)$, where Y_t is the output (GDP), follows an AR(2) model. See *Advanced Macroeconomics* by David Romer (2006, 3rd, pp. 190).

Example: Monthly simple return of Center for Research in Security Prices (CRSP) equal-weighted index

$$R_t = 0.013 + a_t + 0.178a_{t-1} - 0.13a_{t-3} + 0.135a_{t-9}, \quad \hat{\sigma}_a = 0.073$$

Checking: $Q(10) = 11.4(0.122)$ for the residual series a_t .

Implications of the model?

Statistical significance vs economic significance.

In this course, we shall discuss some reasons for the observed serial dependence in index returns. See, for example, Chapter 5 on nonsynchronous trading.

Important properties of a model

- Stationarity condition
- Basic properties: mean, variance, serial dependence
- Empirical model building: specification, estimation, & checking
- Forecasting