

## Financial Time Series

### Homework Assignment #5

**Objective:** To gain experience in using alternative approaches to volatility calculation, neural network, and analysis of high-frequency financial data.

1. The goal of this question is to study the daily log return and its volatility of Alcoa stock. Focus on the period from January 3, 2007 to April 28, 2017. You may download the daily open, high, low, close prices and related information from Yahoo via the `quantmod` package. The tick symbol is `AA`. Note that the 1 for 3 (reverse) stock split on October 6, 2016.

Use the data to construct the (conditional) variance estimates  $\sigma_{i,t}^2$  of Section 3.15.2 of the textbook for  $i = 0, 1, 2, 3, 5,$  and  $6$ . These conditional variances are for the stock price. Take the square-root transformation to obtain the volatility series. Multiple the volatility by  $\sqrt{252}$  to obtain annualized volatility. Obtain mean, median, maximum, and minimum of each of the six annualized volatility series.

2. Again, consider the Alcoa stock, but focus on the volatility of the daily log returns. Use the data and  $n = 63$  to compute the Yang and Zhang (2000) variance estimate  $\hat{\sigma}_{yz}^2$  of Section 3.15.2 of the textbook. Obtain a time plot of the estimated (and annualized) volatility series (square-root of annualized variance). Build a time series model for the log volatility series. Use the fitted model to produce 1-step to 5-step ahead forecasts of log volatility at the forecast origin April 28, 2017.
3. Consider the daily stock returns of Amazon stock (AMZN) and the S&P 500 index from January 4, 2007 to April 28, 2017. You can download the data using `quantmod` from 2007-01-03 to 2017-04-28, then compute the log returns. For instance, `getSymbols('^GSPC', from='2007-01-03', to='2017-04-28')`

Define the direction of price movement of AMZN stock as follows:

$$A_t = \begin{cases} 1 & \text{if } r_t > 0 \\ 0 & \text{otherwise,} \end{cases}$$

where  $r_t$  is the log returns. Similarly, we can define the direction for market movement using the S&P index returns. Denote the direction of market movement by  $M_t$ .

- (a) Fit a linear logistic regression model for  $P(A_t = 1)$  using  $A_{t-1}, A_{t-2}, A_{t-3}, M_{t-1}, M_{t-2}, M_{t-3}$  as input. Based on the model, can past price movements of either the stock or the market predict the future direction of Amazon price movement? Why?
- (b) Use  $(A_{t-i}, M_{t-i})$  for  $i = 1$  and  $2$  to build a 4-3-1 look-forward network with direct link for  $P(A_t = 1)$ . Write down the fitted model.
- (c) Divide the sample into modeling and forecasting subsamples with the latter consisting of the last 250 observations. Use 1-step ahead prediction to compare the previous two models. You can treat  $\hat{A}_t = 1$  if  $P(A_t = 1) > 0.5$ .
4. Consider the tick-by-tick trade data of Walgreens stock from February 6 to February 17, 2017. The data are in the file `taq-wba-feb6to17-2017.txt`.
- Use the data within the normal trading hours only, i.e. from 9:30 am to 4:00 pm Eastern time, to construct a series of intraday 5-minute log returns. If there is no trading within a 5-minute interval, assume that the log return is zero. If there are multiple trades in a 5-minute interval, use the last trade to obtain the price for that interval. Plot the log return series.
  - Are there any serial correlations in the intra-day 5-minute log return series? Use  $Q(10)$  to perform the test.
  - Use 5-minute intraday log returns to compute the realized volatility for each of the trading days.
  - Use 1-minute intraday log returns to compute the realized volatility for each of the trading days.
5. Again, consider the tick-by-tick trade data of Walgreens stock from February 6 to 17, 2017.
- Construct the series of the number of trades within a 5-minute intervals. Use data in the normal trading hours only.
  - Compute the ACF of the constructed time series, say from lag 1 to lag 350. Is there any evidence of diurnal pattern? [No formal test is needed. Simply comment on the ACF plot.]

**Reading assignments:** Sections 4.1, 5.1 to 5.4 of the textbook.

**Deep learning:** If you like to practice on *deep learning*, you can use the daily log returns of Amazon stock in Problem 3 as an exercise. For instance, you can use `darch` or `deepnet` package to predict the stock return using  $(r_{t-i}, s_{t-i})$  for  $i = 1, \dots, 63$  as input, where  $r_t$  and  $s_t$  denote, respectively, the daily log return of Amazon stock and S&P 500 index. You can use the last 600 data points as forecasting period and try several neural networks. For example, use a hidden layer with 100 or 200 nodes in `deepnet`.