STAT 6351: Assignment #4

Notes

- The purpose of this assignment is to analyze the volatility series of daily and monthly asset returns. All models include the mean and volatility equations. You should always perform model checking to confirm the adequacy of a fitted model.

- Use the usual 5% type-I error rate on all tests.

- For daily series, use ten (10) lags in all serial correlations or ARCH-effect tests. For monthly series, use twelve (12) lags.

- An ARMA model includes ARMA(0,0) as special case, meaning that the mean equation contains only the mean of the series.

Questions

1. Consider the daily log returns of Caterpillar stock (CAT) from January 3, 2006 to April 15, 2017. You may download the data using `quantmod`. Let r_t be the log returns, which can be obtained via:

```
getSymbols("CAT",from="2006-01-03",to="2016-04-15")
rt <- diff(log(as.numeric(CAT[,6])))
```

   (a) Are there any serial correlations in r_t? Why?

   (b) Are there any ARCH effects in r_t ? Why?

   (c) Fit a Gaussian ARMA-GARCH model to r_t. Perform model checking, including the normal QQ-plot of the standardized residuals. Write down the fitted model. Is the model adequate?

   (d) Build an ARMA-GARCH model with standardized Student-t innovations for r_t. Perform model checking, including the QQ-plot. Write down the fitted model. Is the model adequate?

   (e) Obtain 1-step to 5-step ahead mean and volatility forecasts using the model in (d).

   (f) Compute 95% 1-step to 5-step interval predictions for r_t using the model in (d).

2. Consider again the daily log returns of the CAT stock of Problem 1. Let $a_t = r_t - \bar{r}$, where $\bar{r}$ is the sample mean of r_t.

   (a) Fit an IGARCH(1,1) model with a constant term in the volatility equation to the a_t series. Write down the fitted model.

   (b) Let σ_t be the fitted volatility of the IGARCH(1,1) model. Show a time-series plot of the estimated volatility series.

   (c) Define the standardized residuals as $\epsilon_t = a_t/\sigma_t$. Is there any serial correlation in ϵ_t? Why?

   (d) Is there any serial correlation in ϵ_t^2? Why?

   (e) Based on the model checking, is the IGARCH model adequate? Obtain 1-step to 4-step ahead volatility forecasts for r_t (the forecast origin is the last data point).

3. Consider the monthly returns of Coke (KO) stock from January 1951 to December 2016. The data are available from CRSP and in the file m-kovw-5116.txt. Obtain the log return series of KO stock, and call it r_t.

   (a) Is the expected value of r_t zero? Why? Is there any serial correlation in r_t? Why? Are there any ARCH effects in r_t? Why?

   (b) Build a GARCH model with Gaussian innovations for r_t. Perform model checking, and write down the fitted model.

   (c) Fit a GARCH model with standardized Student-t innovations to r_t. Perform model checking and write down the fitted model.

   (d) Fit a GARCH model with skew-Student-t innovations to r_t. Based on the fitted model, is the monthly log returns of KO stock skewed? Why?

   (e) Fit a GARCH-M model to r_t. Write down the model. Is the risk premium statistically significant? Why?

   (f) Fit a TGARCH(1,1) model to r_t. Write down the fitted model. Is the leverage effect statistically significant? Why?

4. Consider the monthly returns of the CRSP value-weighted index, including dividends, from 1961 to 2016. The simple returns are also in the file m-kovw-5116.txt (last column). Transform the simple returns of VW to log returns, and call it r_t.

   (a) Find an adequate model for r_t. Perform model checking to justify your model.

   (b) Obtain 1-step to 5-step ahead predictions for r_t and its volatility, at the forecast origin of December 2016.

   (c) Fit a GJR model to r_t. Write down the model. Is the leverage effect statistically significant? Why?

5. Consider again the daily log returns r_t of CAT stock from Problem 1. We would like to study the impact of the VIX index on the volatility of the CAT returns. Let v_t denote the VIX index. You can download the series using quantmod via:

```
getSymbols(``^VIX'',from=``2006-01-03'',to=``2016-04-15'')
vt <- as.numeric(VIX[,6])/100
```

Fit the ARMA-GARCH model with standardized Student-t innovations for r_t as in 1(d), but change the volatility equation to:

$$\sigma_t^2 = \alpha_0 + \gamma v_{t-1} + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2.$$

(a) Test $H_0 : \gamma = 0$ vs. $H_a : \gamma \neq 0$. Compute the test statistic and draw your conclusion.

(b) Obtain the 1-step ahead volatility forecast (for April 15, 2016).