Assignment #4

Note:

1.
2. All tests use the 5% type-I error.
3. For daily series, use ten (10) lags in all serial correlations or ARCH-effect tests. For monthly series, use twelve (12) lags.
4. The purpose of this assignment is to analyze the volatility series of daily and monthly asset returns. **All models include the mean and volatility equations**. You should always perform **model checking** to confirm the adequacy of a fitted model.
5. An ARMA model includes ARMA(0,0) as special case, meaning that the mean equation contains only the mean of the series.

1. Consider the daily log returns of Caterpillar stock (CAT) from January 3, 2006 to April 15, 2017. You may download the data using `quantmod`. Let r_t be the log returns, which can be obtained via

```
rt <- diff(log(as.numeric(CAT[,6])))
```

   (a) Are there any serial correlations in the log return series r_t? Why?

   (b) Are there any ARCH effects in the log return series r_t? Why?

   (c) Fit a Gaussian ARMA-GARCH model to the r_t series. Perform model chekcing, including showing the normal QQ-plot of the standardized residuals. Is the model adequate? Write down the fitted model.

   (d) Build a GARCH model with standardized Student-t innovations for the r_t series. Perform model checking, including the QQ-plot. Is the model adequate? Why?

   (e) Write down the fitted model.

   (f) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted ARMA-GARCH model with standardized Student-t innovations.

(g) Compute the 95% 1-step to 5-step interval predictions of the log return series using standardized student-t innovations.

2. Consider again the daily log returns of the CAT stock of Problem 1.

   (a) Let $a_t = r_t - \bar{r}$, where $\bar{r}$ is the sample mean of r_t. Fit an IGARCH(1,1) model with a constant term in the volatility equation to the a_t series. Write down the fitted model.

   (b) Let σ_t be the fitted volatility of the IGARCH(1,1) model. Show a time-series plot of the estimated volatility series.

   (c) Define the standardized residuals as $\epsilon_t = (r_t - \bar{r})/\sigma_t$. Is there any serial correlation in the standardized residuals? Why?

   (d) Is there any serial correlation in the squares of the standardized residuals? Why?

   (e) Based on the model checking, is the IGARCH model adequate? Obtain 1-step to 4-step ahead volatility forecasts for the r_t series (forecast origin is the last data point).

3. Consider the monthly returns of Coke (KO) stock from January 1951 to December 2016. The data are available from CRSP and in the file `m-kovw-5116.txt`. Obtain the log return series of KO stock.

   (a) Is the expected value of KO log return zero? Why? Is there any serial correlation in the log returns? Why? Is there any ARCH effect in the log returns? Why?

   (b) Build a GARCH model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

   (c) Fit a GARCH model with standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

   (d) Fit a GARCH model with skew-Student-t innovations. Based on the fitted model, is the monthly log returns of KO stock skewed? Why?

   (e) Fit a GARCM-M model to the monthly log returns. Write down the model? Is the risk premium statistically significant? Why?

   (f) Fit a TGARCH(1,1) model to the monthly log returns. Write down the fitted model. Is the leverage effect statistically significant? Why?

4. Consider the monthly returns of the CRSP value-weighted index, including dividends, from 1961 to 2016. The simple returns are also in the file `m-kovw-5116.txt` (last column). Transform the simple returns of VW to log returns.

   (a) Find an adequate model for the monthly VW log return series. Perform model checking to justify your model.

(b) Obtain 1-step to 5-step ahead predictions of the log return of VW index and its volatility at the forecast origin December 2016.

(c) Fit a GJR model to the monthly VW log return series. Write down the model. Is the leverage effect statistically significant? Why?

5. Again, consider the daily log returns of CAT stock of Problem 1. We like to study the impact of VIX index on the volatility of the CAT returns. Let v_t denote the VIX index. You can download the series using `quantmod` via the command

```
getSymbols(''^VIX'',from=''2006-01-03'',to=''2016-04-15'')
```

(a) Use the GARCH model with standardized Student-t innovations as the baseline model. Consider the following volatility equation

$$\sigma_t^2 = \alpha_0 + \gamma v_{t-1} + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2.$$

Test $H_0 : \gamma = 0$ versus $H_a : \gamma \neq 0$. Compute the test statistic and draw your conclusion. Then, use the fitted model to obtain 1-step ahead volatility forecast on April 15, 2016.

Reading assignments: Chapter 3 of the textbook.