STAT 5380 Assignment 4:
Bayes inference, conjugacy, Bayes risk, minimaxity, admissibility

1. TPE Problem 4.1.4.
2. TPE Problem 4.1.6.
3. TPE Problem 4.1.7(a).
   <u>Note:</u> The UMVU estimator of $p(1-p)$ should be:
   $$\delta' = \frac{x(n-x)}{n(n-1)}.$$
4. TPE Problem 4.1.9.
5. TPE Problem 4.1.10.
6. TPE Problem 4.4.1.11.
7. TPE Problem 4.3.6.
8. TPE Problem 4.3.9.
   <u>Note:</u> The natural exponential family and conjugate prior equation numbers should be (3.18) and (3.19), respectively. Also, in (b), the results one needs to show are:
   $$\mathbb{E}A'(\eta) = \mu, \qquad \text{and} \qquad \mathbb{V}[A'(\eta)] = (1/k)\mathbb{E}A''(\eta).$$
9. TPE Problem 4.6.10.
   <u>Note:</u> What is being asked for is to verify that the <u>risk</u> (not Bayes risk) is given by:
   $$p\sigma^2 - \frac{2(p-1)\sigma^4}{\sigma^2+\tau^2} + \left(\frac{\sigma^2}{\sigma^2+\tau^2}\right)^2 \sum_{i=1}^{p} \mathbb{E}(X_i - \bar{X})^2.$$
10. TPE Problem 4.7.5.
11. TPE Problem 5.1.9.
12. TPE Problem 5.1.13.
13. TPE Problem 5.1.25.
14. TPE Problem 5.4.3.
15. Suppose that $\{X_1, \ldots, X_s\}$ are independent with $X_i \sim N(\theta_i, 1)$, and let $\boldsymbol{X} = (X_1, \ldots, X_s)'$ and $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_s)'$. For known $0 < \tau^2 < \infty$, and under the usual squared error loss function
    $$L(\boldsymbol{\theta}, \boldsymbol{d}) = \sum_{i=1}^{s} (\theta_i - d_i)^2,$$
    define the following estimators of $\boldsymbol{\theta}$:
    $$T_1(\boldsymbol{X}) = \boldsymbol{X}, \qquad T_2(\boldsymbol{X}) = \frac{\tau^2}{1+\tau^2}\boldsymbol{X}.$$
    Note that by Remark 4.4.5 of notes, we know that T_1 is not admissible.
    (a) Show that T_2 is admissible.
    (b) Show that T_1 is minimax. [Hint: Theorem 4.2.8 of notes with $\Lambda_n \sim$ iid $N(0, n)$.]