STAT 5379 Assignment 3

(Covered in Homeworks 10–12)

Mathematical concepts and derivations

1. (S&S Problem 3.33.) Fit an ARIMA(p, d, q) model to the global temperature data `globtemp`, performing all of the necessary diagnostics. After deciding on an appropriate model, forecast (with limits) the next 10 years. Comment.

2. (S&S Problem 4.6.) Consider the AR(1) model:

$$X_t = \phi X_{t-1} + Z_t, \qquad Z_t \sim \text{IID}(0, \sigma^2), \quad |\phi| < 1.$$

   (a) Show that the spectral density of X_t is:

$$f(\omega) = \frac{\sigma^2}{1 + \phi^2 - 2\phi\cos(2\pi\omega)}.$$

   (b) By showing that the inverse transform of $\gamma(h)$ is the spectral pdf derived in part (a), verify that:

$$\gamma(h) = \frac{\sigma^2 \phi^{|h|}}{1 - \phi^2}.$$

3. (S&S Problem 4.7.) Suppose we observe a series containing a signal that has been delayed by some unknown time D, i.e.,

$$X_t = s_t + A s_{t-D} + \eta_t,$$

   where s_t and η_t are stationary and independent with zero means and spectral densities $f_s(\omega)$ and $f_\eta(\omega)$, respectively. The delayed signal is multiplied by some unknown constant A. Show that the spectral density of X_t is:

$$f_x(\omega) = [1 + A^2 + 2A\cos(2\pi\omega D)] f_s(\omega) + f_\eta(\omega).$$

4. S&S Problem 6.8.

Numerical work and simulations

5. (S&S Problem 4.25.) Consider the two processes

$$X_t = W_t, \qquad Y_t = \phi X_{t-D} + V_t,$$

   where W_t and V_t are independent white noise processes with common variance σ^2, ϕ is a constant, and D is a fixed integer (time delay).

   (a) Compute the coherency between X_t and Y_t.

   (b) Simulate $n = 1024$ normal observations from X_t and Y_t for $\phi = 0.9$, $\sigma^2 = 1$, and $D = 0$. Then estimate and plot the coherency between the two simulated series for the following values of L and comment: (i) $L = 1$, (ii) $L = 3$, (iii) $L = 41$, (iv) $L = 101$.

6. S&S Problem 6.1.

Applications

7. B&D Problem 10.2.

8. B&D Problem 11.5.

9. (S&S Problem 4.9.) Compute the periodogram for the `sunspotz` data (Figure 4.22). Identify the predominant frequencies (and corresponding periods), and construct 95% confidence intervals for the spectral density at those frequencies.

10. (Variation on S&S Problem 4.14.) Estimate the spectral density of the `sunspotz` data using two nonparametric smoothing methods: (i) function "parzen.wge" in package `tswge`, and (ii) any other smoothing method. With the periodogram as a guide, which method seems to give a more accurate result?

11. (S&S Problem 4.19.) Fit an autoregressive spectral estimator to the `sunspotz` data using a model selection method of your choice. Compare the result of this parametric method with the 3 nonparametric methods in the above Problems.

12. S&S Problem 6.6.