STAT 5374

Hwk #6

1. Suppose that a population of married couples have heights in inches X for the wife and Y for the husband. Suppose that $(X, Y)^T$ has a bivariate normal distribution with parameters $\mu_X = 66, \mu_Y = 70, \sigma_X = 2.5, \sigma_Y = 2.7$, and $\rho = .3$. What is the probability that the husband is taller than his wife? Does the probability increase or decrease with ρ?

2. Let $X_1, \ldots, X_{n_1}$ and let $Y_1, \ldots, Y_{n_2}$ be independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. Find the distributions of the following random variables. In each case, name the distribution and give its parameters.

   a. $\frac{1}{\sigma_1^2}\sum_{i=1}^{n_1}(X_i - \mu_1)^2$, and $\frac{1}{\sigma_1^2}\sum_{i=1}^{n_1}(X_i - c_i)^2$, for arbitrary constants $c_1, \ldots, c_{n_1}$.

   b. $\frac{n_1}{\sigma_1^2}(\bar{x} - \mu_0)^2$, $\frac{n_1}{\sigma_1^2}(\bar{x} - \mu_1)^2$ (μ_0 here is an arbitrary constant).

   c. $n_1(\bar{x} - \mu_1)^2/\sigma_1^2 + n_2(\bar{y} - \mu_2)^2/\sigma_2^2$.

   d. $\frac{n_2 - 1}{n_1}[\sum_i^{n_1}(x_i - \mu_0)^2]/[\sum_i^{n_2}(y_i - \bar{y})^2]$. Here, assume $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and μ_0 is an arbitrary constant.

3. Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be a random sample from the bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$. Find a constant K so that

$$T = K\frac{(\bar{X} - \bar{Y}) - \delta}{\{\sum_{i=1}^{n}[(X_i - Y_i) - (\bar{X} - \bar{Y})]^2\}^{1/2}} \sim t(m, \theta).$$

Express m and θ as functions of the parameters and the constant δ. (*Hint:* Let $D_i = X_i - Y_i$. Express T as a function of the D_i's.)

4. Problem **5.27** in our text.

5. Problem **5.30** in our text.