STAT 5374

Hwk #4

#1. Let

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -1 & 1 \\ 3 & -3 & 4 \end{pmatrix}, \quad \mathbf{A}^- = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -2 & 0 \\ 1 & -1 & 0 \end{pmatrix}.$$

Use these matrices to illustrate the following general result:

For $\mathbf{X}$ an $n \times k$ matrix of rank r and $\mathbf{X}^-$ its generalized inverse, then

$$\operatorname{tr}(\mathbf{X}^-\mathbf{X}) = \operatorname{tr}(\mathbf{X}\mathbf{X}^-) = \operatorname{rank}(\mathbf{X}^-\mathbf{X}) = \operatorname{rank}(\mathbf{X}\mathbf{X}^-) = \operatorname{rank}(\mathbf{X}).$$

#2. Compare the rank of the augmented matrix with the rank of the coefficient matrix for each of the following systems of equations. Find solutions where they exist.

a.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ -x_1 - x_2 &= 1 \\ -x_1 + x_2 + 2x_3 &= 9 \end{aligned}$$

b.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ -x_1 - x_2 &= 1 \\ x_2 + x_3 &= 9 \end{aligned}$$

c.

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 1 \\ -x_1 - x_2 + x_4 &= 2 \\ x_2 + x_3 - x_4 &= 1 \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= 1 \end{aligned}$$

#3. For the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

and its generalized inverses

$$\mathbf{A}_1^- = \frac{1}{9}\begin{pmatrix} 1 & -5 & -4 \\ 2 & -1 & 1 \\ 1 & 4 & 5 \end{pmatrix}, \quad \text{and} \quad \mathbf{A}_2^- = \begin{pmatrix} -1 & -2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

show that $\mathbf{A}\mathbf{A}_1^-\mathbf{A} = \mathbf{A}\mathbf{A}_2^-\mathbf{A} = \mathbf{A}$.

#4. For the matrix $\mathbf{A}$ of problem 9, show that the generalized inverse of $\mathbf{A}$ given by

$$\mathbf{A}_3^- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 2 & 0 \end{pmatrix}$$

is a generalized inverse of $\mathbf{A}$ can be derived by the 5 step algorithm given by Corollary 1 following Theorem 2.8B of our textbook (Rencher & Schaalje.).

#5. Problem 2.31.

#6. Problem 2.46.