STAT 5374 Hwk #3

1. For the example on p.42 of notes.
(a) Verify that $V_1^{\perp}$, $V_2^{\perp}$, $V^{\perp}$ are as given.
(b) " " V_1 is not orthogonal to V_2.
(c) Verify the statement $P_V \neq P_{V_1} + P_{V_2}$ by computing $p(\underline{y}|V)$, $p(\underline{y}|V_1)$, $p(\underline{y}|V_2)$ for $\underline{y} = [1,2,3,4]^T$, as suggested.
(d) Show that $P_V \neq P_{V_1} + P_{V_2}$ by computing P_V, P_{V_1}, P_{V_2} directly.

2. Verify the result $C(\mathbf{X}^T\mathbf{X}) = C(\mathbf{X}^T)$ for

$$\mathbf{X} = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 1 & 1 \end{pmatrix}.$$

3. For each of the following matrices or quadratic forms, determine whether it is positive definite, positive semidefinite, or neither:

a. $Q(x_1, x_2, x_3) = 12x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 - 10x_1x_3 + 4x_2x_3$.

b.

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 4 \end{pmatrix}$$

c.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & -2 \end{pmatrix}$$

4. For $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ find a matrix $\mathbf{B}$ such that $\mathbf{A} = \mathbf{B}\mathbf{B}^T$.

5. Problem 2.35.

6. Problem 2.39.

7. Problem 2.71 (a, b, c), and also these additional parts:

d. For $k = 3$, verify the property $\mathbf{A}^k\mathbf{v} = \lambda^k\mathbf{v}$ for each eigen-pair of $\mathbf{A}$.

e. Find $\mathrm{tr}(\mathbf{A})$ and $|\mathbf{A}|$ and verify that $\mathrm{tr}(\mathbf{A}) = \sum_{i=1}^3 \lambda_i$ and $|\mathbf{A}| = \prod_{i=1}^3 \lambda_i$ where $\lambda_1, \lambda_2, \lambda_3$ are the eigenvalues