Symmetry and symmetrization in geometry, analysis, and PDE's

MATH 5342-4 Spring 2013
Course instructor: Dr. Alexander Solynin
MA 114 MWF 2:00 – 2:50 pm
Office hours: MWF 10:00-11:00 or by appointment
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TEXT: Your lecture notes.
Complementary texts: (1) Multivalent functions. Second edition. Cambridge Tracts in Mathematics, 110. *Cambridge University Press, Cambridge,* 1994, by W. K. Hayman.
(2) Isoperimetric Inequalities in Mathematical Physics. Annals of Mathematics Studies, no. 27, *Princeton University Press, Princeton, N. J.,* 1951 by Pólya, G. and Szegö, G.
(3) Symmetrization in the geometric theory of functions of a complex variable, *Russian Math. Surveys* **49** (1994), no. 1, 1—79 by Dubinin, V. N.

- Course Description: This course is about extremal problems arising in geometry, complex analysis, and mathematical physics. In particular, we will discuss different generalizations of the classical isoperimetric problem saying that the disk has the maximal area among all planar regions having a fixed perimeter. In mathematical physics, this will lead to several nice inequalities between geometrical characteristics of a region and some of its functional characteristics such as capacity, torsional rigidity, and principal frequency.

  In complex analysis, we will study several extremal problems on the behavior of conformal mappings and univalent functions.

  Usually, configurations extremal for the considered isoperimetric inequalities are highly symmetric. The method of symmetrization, which often allows reaching the desired symmetry, also will be discussed. In particular, we plan to study geometric transformations such as Steiner's symmetrization, circular symmetrization, and polarization, as well as their applications.

An undergraduate course of partial differential equations, a course of real analysis, and one semester course of complex variables will be very useful.

Learning Outcomes: Upon completion *Symmetrization course* students will master concepts and theories of Steiner Symmetrization, Circular Symmetrization, Polarization, Dissymmetrization.

The main emphasis will be given to applications of these transformations to inequalities in Mathematical Physics and to Extremal Problems for analytic functions and conformal mappings.

Methods for Assessment of Learning Outcomes: The expected learning outcomes for the course will be assessed through graded activities and ungraded activities. The graded activities include exams, homework, quizzes, and research papers. The ungraded activities will be used to monitor your progress. A variety of these ungraded assessment techniques may be employed, including problems to be completed during class, direct questioning of students, answering students questions in class, one-minute classroom assessment techniques, and discussions during office hours.

Students with Disabilities: Any student who because of a disability may require special arrangements in order to meet course requirements should contact the instructor as soon as possible to make any necessary accommodations. Students should present appropriate

verification from AccessTECH. No requirement exists that accommodations be made prior to completion of this approved university procedure.

Absence due to religious observance: The Texas Tech University Catalog states that a student who is absent from classes for the observance of a religious holy day will be allowed to take an examination or complete an assignment scheduled for that day within a reasonable time after the absence. Notification must be made in writing and delivered in person no later than 15th class day of the semester.

Absence due to officially approved trips: The Texas Tech University Catalog states that the person responsible for a student missing class due to a trip should notify the instructor of the departure and return schedule in advance of the trip. The student may not be penalized and is responsible for the material missed.

Academic Integrity: It is the aim of the faculty of Texas Tech University to foster a spirit of complete honesty and a high standard of integrity. There will no tolerance for cheating or plagiarism. Texas Tech University policies will be enforced in such cases.

STUDENT EVALUATION:

- A student has a choice either take the Final which is a comprehensive, course wide exam or to work on his/her research project related to this topic and present results upon completion.

♦ **May 10 FINAL EXAMINATION 1:30p.m. – 4:00 p.m. 60 pts**

This exam is scheduled before the semester begins.

Students should **eliminate any conflicts NOW.**

- **MIDTERM EXAM/PRESENTATION: 40 pts**
- **HOMEWORK** (Problems and Exercises): **40 pts**
- **MAXIMAL TOTAL: 140 pts**
- **Optional Research Projects:** Research projects are optional and may be used to substitute the Final Exam.

GRADING PROCEDURE:

A - 90 - 100%

B - 80 - 89%

C - 70 -79%

D - 60 - 69%

F - ≤ 59%

Important Dates:

January 21 - Martin Luther King Jr. Day.
March 9-17 - Spring Vacation.
March 27 - Final day to drop a course.
April 1 - No Classes.
May 7 - Last Day of classes.

Friday, May 10 - 1:30 – 4:00 pm. Final Exam (Examinations will be given in the rooms in which the individual classes have been meeting unless otherwise announced**).**

May 20 - Final Grades Due via Web.