Germain's Variational Problems and Noether's Symmetries

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Emmy Noether Day

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Historical Background

Ernst Chladni (1787)

Chladni Plates
(https://www.youtube.com/watch?v=9uEeADQN8Jo)

The Principle of Least Action

Statement

Any change in nature takes place using the minimum amount of required energy.

- G. Leibniz (1705)
- L. Euler (1744)
- P. L. Maupertuis (1744-1746)

What is the energy for Chladni's experiment?

Marie-Sophie Germain (1776-1831)

A French woman and self-taught mathematician.
She initiated the Theory of Elasticity.

Solution

The energy involves the curvature of the surface modeling the plate.

What is curvature?

What is curvature?

- Gaussian curvature:

$$K = \kappa_1 \kappa_2 \, .$$

- Mean curvature (coined by Germain):

$$H = \frac{\kappa_1 + \kappa_2}{2} \, .$$

Germain's Variational Problems

Germain proposed to find minima of

$$\mathcal{W}[\Sigma] := \int_{\Sigma} H^2 \, d\Sigma \, .$$

- Now know as: Willmore Energy.
- Different choices are also interesting: area, Helfrich energy,...

Minimize $\mathcal{W}$ and similar energies is a nightmare!

Techniques arising from the Calculus of Variations are used, but...

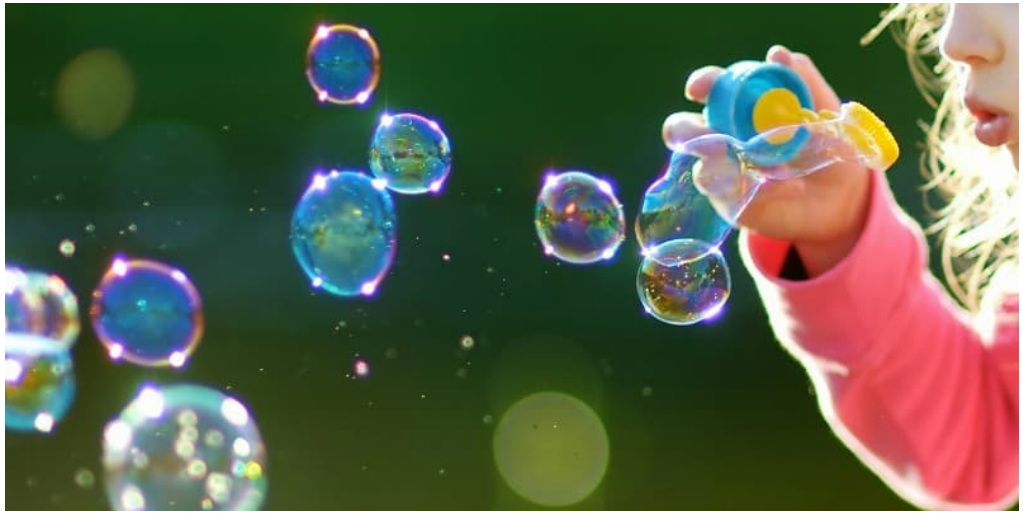
Noether's Symmetries

Informal Statement

If an energy possesses some symmetries, then there are associated conserved quantities.

Emmy Noether (1882-1935)

Soap Bubbles



(https://www.youtube.com/watch?v=Krtttg4HZE)

THE END

Thank You!