

MATH CIRCLE TTU

Number Theory

Induction Axiom



Adding “Easy” Sequences

1. What is the sum of the first n natural numbers?

$$1 + 2 + 3 + 4 + \cdots + (n - 1) + n = ?$$

2. What is the sum of the first n odd natural numbers?

$$1 + 3 + 5 + 7 + \cdots + (2n - 3) + (2n - 1) = ?$$

3. What is the sum of the first n even natural numbers?

$$2 + 4 + 6 + 8 + \cdots + (2n - 2) + (2n) = ?$$

Adding “Not-That-Easy” Sequences

4. Follow the Induction Process to show that

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Hint (the following are the steps for the Induction Process):

- (i) Check it for $n = 1$ (also, if you do not trust it, check it for $n = 2, 3, \dots$).
- (ii) Assume it is true for $n - 1$. That is, assume that

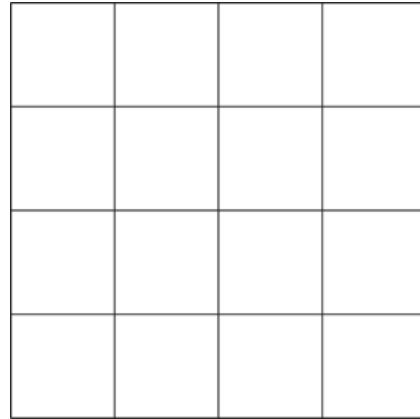
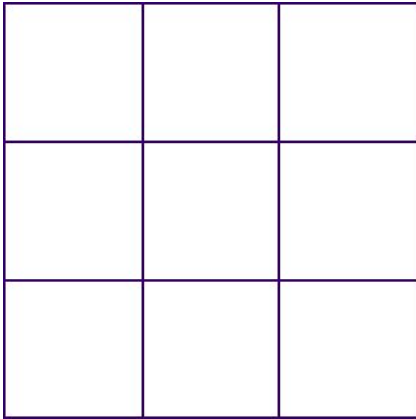
$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + (n-1)^2 = \frac{(n-1)n(2n-1)}{6}.$$

- (iii) Using the identity of the previous step, prove the original formula.

5. What is the sum of the squares of the first n even numbers?

$$2^2 + 4^2 + 6^2 + 8^2 + \cdots + (2n)^2 = ?$$

Magic Squares



A magic square is a $n \times n$ 'big' square that has the numbers $1, 2, 3, \dots, n^2$ in the 'small' squares, in such a way that the sum of the numbers in each row, each column, and both main diagonals are the same.

6. Before solving it, what is the constant sum for a 3×3 magic square?
7. And, for a 4×4 ?
8. And, in general, for a $n \times n$ magic square?
9. Solve the 3×3 magic square.
10. (Homework) Solve the 4×4 magic square.