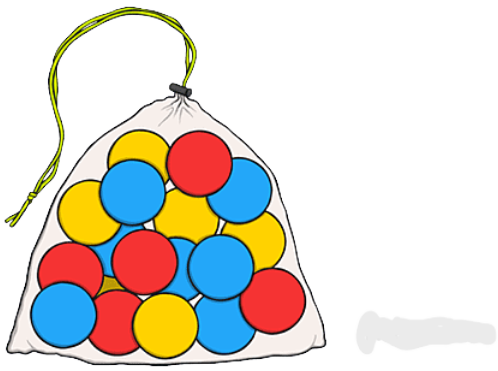
MATH CIRCLE TTU

Discrete Mathematics

Counting Techniques



Permutations

Definition. Consider a set with 'n' different elements. An *ordered* subset is called a **permutation**. (There exist permutations with repetition.)

Question. How many ways are there to distribute the medals (Gold, Silver, and Bronze) in the Olympic games among 10 athletes?

Question. How many telephone numbers of 10 digits may exist?

Combinations

Definition. Consider a set with 'n' different elements. An *unordered* subset is called a **combination**. (There exist combinations with repetition.)

Question. Among 10 flavors of ice cream, how many possible variations do we have to get three scoops of different flavors? And, if we can repeat the flavors?

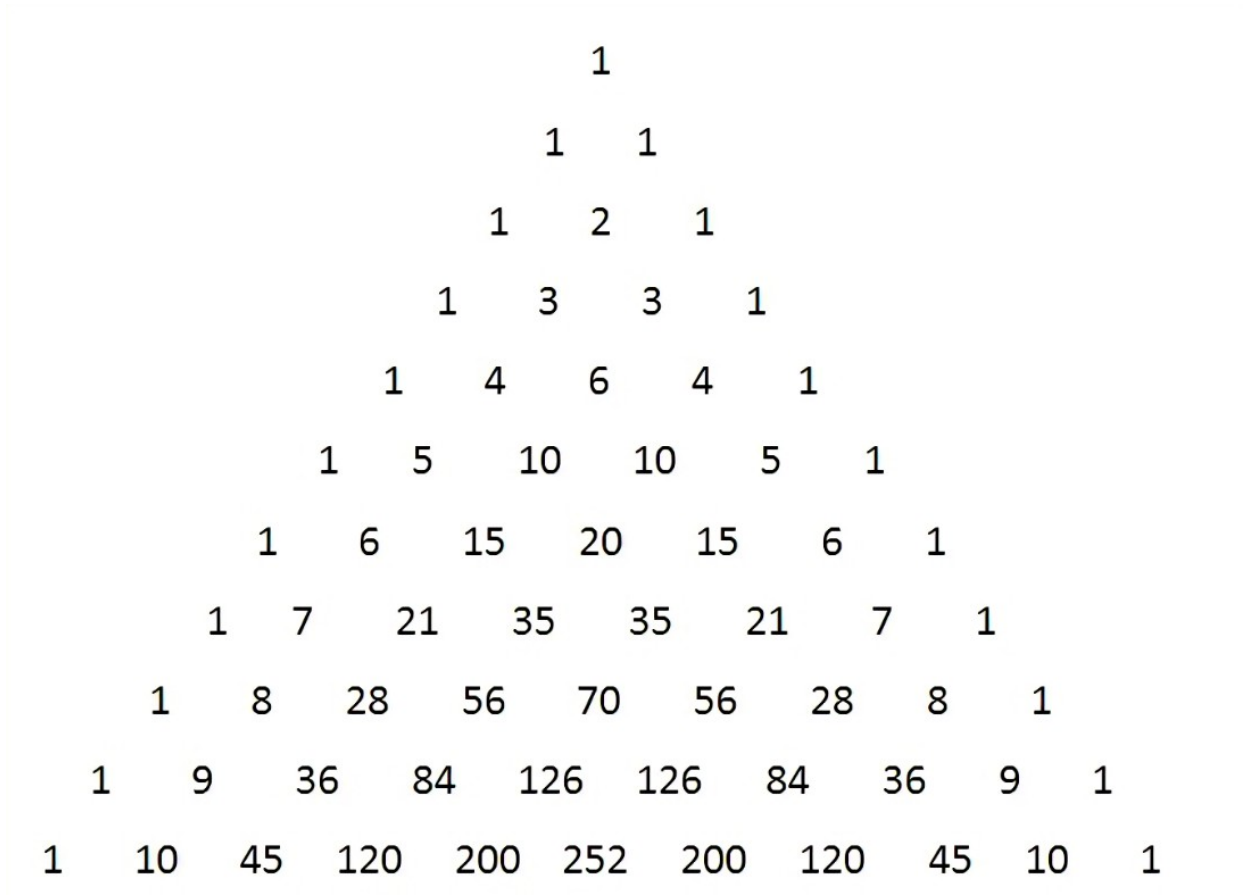
Combinatorial Numbers

Math Fact. The number of combinations of size k that can be formed from n elements (without repetition) is:

$$C_{k,n} = \binom{n}{k}.$$

The number $C_{k,n}$ is called a **combinatorial number**.

Pascal's Triangle



Binomial Theorem

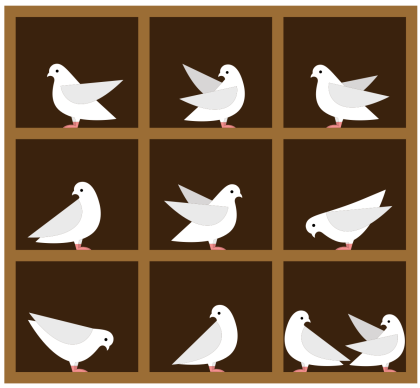
Question. Expand the following expressions:

- $(a+b)^0 =$
- $(a+b)^1 =$
- $(a+b)^2 =$
- $(a+b)^3 =$
- $(a+b)^4 =$
- $(a+b)^5 =$
- $(a+b)^6 =$
- $(a+b)^7 =$

Question. Do the coefficients of the above expanded polynomials sound familiar to you?

The Pigeonhole Principle

Statement. If we try to put a fixed number of pigeons into a container with less holes than pigeons, then at least one hole must have at least two pigeons.



Question. If a Martian has red, blue, yellow, and black socks in a drawer, what is the minimum number of socks that the Martian must pull out of the drawer to guarantee he has a pair?

Question. Explain why there must exist at least two people in Lubbock with the same number of hairs on their heads. (Useful information: nobody can have more than $200,000$ hairs on their head and the population of Lubbock is $269,886$.)

The Birthday Problem: A New Paradox

Question. The birthday problem asks for the probability that, among 'n' people at least two share a birthday (we do not consider February 29). How many people are needed for that the probability is bigger than 50%?

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