MATH 5355, Spring 2017 **Matlab Lab** Name:

Simulating Continuous-time Markov Chains

1. Consider the CTMC SIR epidemic model with no births and deaths. Download the code from the course website.
   (a) For $\beta = 0.5$, $\gamma = 0.25$, and $N = 100$ run the code to simulate three sample paths. What is $\mathcal{R}_0$? For each stochastic realization how long did the epidemic last?
   (b) Modify the code to calculate the average duration of the epidemic over 1000 realizations.
   (c) Estimate the average duration of the epidemic if $\beta = 0.75$ and $\mathcal{R}_0 = 7.5$.
   (d) How does the duration of infection relate to various $\mathcal{R}_0$? For a fixed $\beta = 0.5$ investigate a range of $\gamma \in [0.01, 3]$. Plot $\mathcal{R}_0$ vs the estimated length of epidemic.

2. Consider the CTMC two species competition model (Section 7.7 in the book) with the following transition probabilities.

$$\text{Prob}\{\Delta X(t) = i, \Delta Y(t) = j | (X(t), Y(t))\} = \begin{cases} a_{10}X(t)\Delta t + o(\Delta t), & (i,j) = (1,0) \\ a_{20}Y(t)\Delta t + o(\Delta t), & (i,j) = (0,1) \\ X(t)[a_{11}X(t) + a_{12}Y(t)]\Delta t + o(\Delta t), & (i,j) = (-1,0) \\ Y(t)[a_{21}X(t) + a_{22}Y(t)]\Delta t + o(\Delta t), & (i,j) = (0,-1) \\ 1 - X(t)[a_{11}X(t) + a_{12}Y(t)]\Delta t & \\ \quad -Y(t)[a_{21}X(t) + a_{22}Y(t)]\Delta t + o(\Delta t), & (i,j) = (0,0) \\ o(\Delta t), & \text{otherwise} \end{cases}$$

   Where parameters values are $a_{10} = 2, a_{11} = 0.03, a_{12} = 0.02, a_{20} = 1.5, a_{21} = 0.01, a_{22} = 0.04$ and $X(t_0) = 50, Y(t_0) = 25$ where $t_0 = 0$. Download the code from the course website.
   (a) Simulate sample paths for time from 0 to 5.
   (b) Simulate sample paths for time from 0 to 50.

3. Consider the CTMC two species predator-prey model (Section 7.8 in the book) with the following transition probabilities.

$$\text{Prob}\{\Delta X(t) = i, \Delta Y(t) = j | (X(t), Y(t))\} = \begin{cases} a_{10}X(t)\Delta t + o(\Delta t), & (i,j) = (1,0) \\ a_{21}X(t)Y(t)\Delta t + o(\Delta t), & (i,j) = (0,1) \\ a_{12}X(t)Y(t)\Delta t + o(\Delta t), & (i,j) = (-1,0) \\ a_{20}Y(t)\Delta t + o(\Delta t), & (i,j) = (0,-1) \\ 1 - X(t)[a_{10} + a_{12}Y(t)]\Delta t & \\ -Y(t)[a_{20} + a_{21}X(t)]\Delta t + o(\Delta t), & (i,j) = (0,0) \\ o(\Delta t), & \text{otherwise} \end{cases}$$

   where parameter values are $a_{10} = 1, a_{20} = 1, a_{12} = 0.02, a_{21} = 0.01$ and $X(t_0) = 120, Y(t_0) = 40$ where $t_0 = 0$

(a) Write a Matlab program to simulate sample paths for the stochastic predator-prey model for time from 0 to 20.

(b) Plot the sample paths for $X(t)$ and $Y(t)$ vs time on the same graph.

(c) Plot the sample paths on a phase plane digram, $X(t)$ vs. $Y(t)$.

(d) Describe the dynamics of the populations over time.