Another group associated with a Hopf algebra $H$ arises as follows: The convolution product of two algebra homomorphisms $H \to F$ is again an algebra homomorphism, and if $\varphi\colon H \to F$ is an algebra homomorphism then so is $\varphi \circ \iota$, and in that case $\varphi$ and $\varphi \circ \iota$ are inverses with respect to the convolution product.
Thus, the algebra homomorphisms $H \to F$ form a group under the convolution product.

**Examples:** For $FG$, the algebra homomorphisms correspond to group homomorphisms $G \to F^*$, with point-wise multiplication.

For $F[x]$, the algebra homomorphisms correspond to the elements in $F$, with addition.

For $F^{(G)}$, the algebra homomorphisms are the coordinate maps for the basis $(e_\sigma)_\sigma$, forming a group isomorphic to $G$.

For $F[x, y, y^{-1}]$, an algebra homomorphism must send $x$ to 0 and $y$ to a non-zero scalar, giving us the group $F^*$.

For $F \oplus F \oplus F(\sqrt{D})$, there are two algebra homomorphisms, giving us the group $C_2$.

An important class of examples comes from algebraic geometry:

Let $G \le \mathrm{GL}_n(F)$ be a *linear algebraic group* over $F$, i.e., a Zariski-closed subgroup of $\mathrm{GL}_n(F)$.
Then the coordinate ring $A(G)$ for $G$ has the form

$$A(G) = F[X, 1/\det X]$$

where $X$ is a *generic* element of $G$, i.e., it maps exactly to the elements of $G$ under the algebra homomorphisms $A(G) \to F$.

To formally encode the group structure of $G$ on $A(G)$, we make $A(G)$ into a Hopf algebra by

$$\begin{aligned}\mu(X) &= (X \otimes 1)(1 \otimes X), \\ \varepsilon(X) &= I, \quad \text{and} \\ \iota(X) &= X^{-1}\end{aligned}$$

The convolution product of two algebra homomorphisms then corresponds to the product of the images of $X$, so the group of algebra homomorphisms is isomorphic to $G$.

A finite group is always linear algebraic, and the corresponding Hopf algebra is $A(G) = F^{(G)}$.

If $F$ is infinite, the linear algebraic group

$$\left\{ \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \;\middle|\; a \in F \right\} \simeq (F, +)$$

has coordinate ring $F[x]$, with Hopf algebra structure as before.

If $F = \mathbb{F}_q$ is finite, the coordinate ring is

$$\mathbb{F}_q[x]/(x^q - x) \simeq \mathbb{F}_q^{(\mathbb{F}_q)}$$

**Example:** Let $F$ be an infinite field. For the linear algebraic group

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \;\middle|\; a, b \in F,\ a \neq 0 \right\} \simeq F \rtimes F^*$$

we get $A(G) = F[x, y, 1/y]$ with

$$\mu(x) = x \otimes 1 + y \otimes x, \quad \mu(y) = y \otimes y,$$
$$\varepsilon(x) = 0, \quad \varepsilon(y) = 1,$$
$$\iota(x) = -x/y, \quad \iota(y) = 1/y$$

**Example:** Again, let $F$ be infinite, and of characteristic $\neq 2$. For the special orthogonal group

$$\mathrm{SO}(2) = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \,\middle|\, a, b \in F,\ a^2 + b^2 = 1 \right\}$$

we get $A\big(\mathrm{SO}(2)\big) = F[c, s = \sqrt{1 - c^2}]$ with

$$\begin{aligned} \mu(c) &= c \otimes c - s \otimes s, \\ \mu(s) &= c \otimes s + s \otimes c, \\ \varepsilon(c) &= 1, \quad \varepsilon(s) = 0, \\ \iota(c) &= c, \quad \iota(s) = -s \end{aligned}$$

The group-like elements in $A(G)$ are those giving group homomorphisms $G \to F^*$.